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Research Article

On Hamming Eigenvalues and Hamming Energy of Graphs*

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Abstract

The Hamming distance between two strings of the same length is the count of positions at which their corresponding symbols are different. For a graph G of order n, the Hamming matrix is $H(G) = [h_{ij}]_{n\times n}$, where h_{ij} equals the Hamming distance between $s(v_i)$ and $s(v_j)$, and $h_{ii} = 0$; here, $s(v_i)$ represents the string corresponding to the i-th row of the incidence matrix of G. The Hamming energy was recently introduced in analogy with adjacency energy. In this paper, it is shown that, except for edgeless graphs and K_2, the Hamming matrix has exactly one positive eigenvalue; as a result, a general expression for the Hamming energy is derived. This result shows that the study of the Hamming energy reduces to the analysis of the largest eigenvalue of the Hamming matrix. These results extend several earlier findings on the Hamming matrix in the literature.

Keywords: adjacency matrix; Hamming matrix; Hamming distance; graph energy; Hamming energy.

2020 Mathematics Subject Classification: 05C50, 05C12.

1. Introduction

Let G be a simple undirected graph with n vertices and m edges. Let its vertex set be $V(G) = \{v_1, v_2, \ldots, v_n\}$ and its edge set be $E(G) = \{e_1, e_2, \ldots, e_m\}$. Two vertices v_i and v_j are said to be adjacent if there exists an edge connecting them, in which case we write $v_i \sim v_j$; otherwise, they are non-adjacent, denoted by $v_i \nsim v_j$. The degree of vertex $v_i \in V(G)$ is the number of edges incident with it, denoted by $\deg_G(v_i)$. A graph in which all vertices have the same degree (say r) is called a regular graph (r-regular). Graph energy (adjacency energy) was first introduced in 1978 [7]. It is defined as

$$\mathcal{E}(G) = \sum_{i=1}^{n} |\lambda_i|,$$

where $\lambda_1, \lambda_2, \ldots, \lambda_n$ denote the eigenvalues of the adjacency matrix associated with the graph G [5]. Nikiforov [10] extended this concept to arbitrary matrices (not necessarily square) and defined their energy as the sum of their singular values. Although graph energy originated in molecular chemistry, it has applications in a variety of fields, for instance, see the survey paper [8]. The incidence matrix of the graph G with n vertices and m edges is a matrix $B(G) = [b_{ij}]_{n\times m}$, with $b_{ij} = 1$ if and only if vertex v_i is incident with edge e_j, and 0 otherwise.

The study of Hamming distances between the strings corresponding to the rows of the incidence matrix was initiated by Ramane et al. [11]. Using these Hamming distances, recently, Vučićević et al. [15] introduced the Hamming matrix of a graph and studied its energy. They obtained the Hamming energy for certain classes of graphs, provided some bounds on it, tested its predictive potential and showed that it is useful. Redžepović et al. [13] tested the sensitivity of the Hamming energy on some chemical graphs and showed that it is exceptionally sensitive compared with graph energy as well as other eigenvalue-based graph invariants. Recent work on the Hamming matrix can be found in [3, 4, 12–15]. The Hamming distance serves as a fundamental metric in computer science, particularly in the design and analysis of error-detecting and error-correcting codes.

In the present work, we determine the signs of the Hamming eigenvalues and derive a general expression for the Hamming energy in terms of the Hamming spectral radius.

*This article is dedicated to Prof. H. B. Walikar on the occasion of his 75th birthday (18 March 2026).

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Shahin

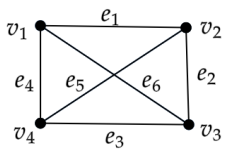


Figure 2.1: The complete graph K_4 with labeled vertices and edges.

2. Preliminaries

Let $(\mathbb{Z}_2, \oplus_2)$ be the additive group, where $\mathbb{Z}_2 = \{0, 1\}$ and $\oplus_2$ denotes addition modulo 2. Let the row of the incidence matrix of a graph of order n and size m corresponding to a vertex v_i be denoted by $s(v_i)$. Then, $s(v_i) = (b_{i1}, b_{i2}, \ldots, b_{im})$ is a vector in the set $\mathbb{Z}_2^m = \mathbb{Z}_2 \times \mathbb{Z}_2 \times \cdots \times \mathbb{Z}_2$ (m times) represented by $s(v_i) = b_{i1}b_{i2}\cdots b_{im}$. The Hamming distance between two strings of equal length is defined as the number of positions at which the corresponding entries differ. Therefore,

$$H_d(s(v_i), s(v_j)) = \sum_{k=1}^{m} (b_{ik} \oplus_2 b_{jk}).$$

For example, if $s(u) = 10110$ and $s(v) = 11010$, then $H_d(s(u), s(v)) = 2$.

The Hamming index of a graph G, denoted by $H_B(G)$, is defined as the total sum of the Hamming distances over all pairs of strings derived from the incidence matrix $B(G)$ of G [11]. Alternatively,

$$H_B(G) = \sum_{\{v_i, v_j\} \subseteq V(G)} H_d(s(v_i), s(v_j)).$$

The Hamming matrix of a graph G is a square matrix $H(G) = [h_{ij}]$ of order n where $h_{ij} = H_d(s(v_i), s(v_j))$. The eigenvalues of $H(G)$ are called Hamming eigenvalues and are denoted by $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_n$ and the largest Hamming eigenvalue μ_1 is called the Hamming spectral radius. The sum of the absolute values of the eigenvalues of $H(G)$ is called the Hamming energy of G and is denoted by $HE(G)$ [15]. Thus, if $\mu_1, \mu_2, \ldots, \mu_n$ are the Hamming eigenvalues of G, then

$$HE(G) = \sum_{i=1}^{n} |\mu_i|.$$

Example 2.1. *For the graph shown in Figure 2.1, the incidence matrix is*

$$B(K_4) = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

and the corresponding Hamming matrix is

$$H(K_4) = \begin{bmatrix} 0 & 4 & 4 & 4 \\ 4 & 0 & 4 & 4 \\ 4 & 4 & 0 & 4 \\ 4 & 4 & 4 & 0 \end{bmatrix}.$$

Therefore, the Hamming index is $H_B(K_4) = 4 + 4 + 4 + 4 + 4 + 4 = 24$. *The eigenvalues of the Hamming matrix are* $\mu_1 = 12$, $\mu_2 = -4$, $\mu_3 = -4$ *and* $\mu_4 = -4$. *Hence, the Hamming energy is*

$$HE(K_4) = \sum_{i=1}^{4} |\mu_i| = 24.$$

Theorem 2.1. [11] *Let v_i and v_j be distinct vertices of G. Then*

$$H_d(s(v_i), s(v_j)) = \begin{cases} \deg_G(v_i) + \deg_G(v_j) - 2, & \text{if } v_i \sim v_j \\ \deg_G(v_i) + \deg_G(v_j), & \text{if } v_i \nsim v_j. \end{cases}$$

Theorem 2.2. [15] *For a graph G of order n with Hamming matrix $H(G) = [h_{ij}]_{n \times n}$, the Hamming index is*

$$H_B(G) = \frac{1}{2} \sum_{i,j} h_{ij}.$$

Theorem 2.3. [9] (Gershgorin's Theorem) *Let $P = [p_{ij}]$ be an $n \times n$ matrix. If λ is an eigenvalue of P, then there exists an index $k \in N = \{1, 2, \ldots, n\}$ such that*

$$|\lambda - p_{kk}| \leq \sum_{\substack{j \in N \\ j \neq k}} |p_{kj}|.$$

Let $W = \{y = (y_1, y_2, \ldots, y_n) \in \mathbb{R}^n : \sum_{i=1}^{n} y_i = 0\}$. A real square symmetric matrix M of order n is called conditionally negative definite if $y^T M y \leq 0$ for every $y \in W$.

Proposition 2.1. [2] *Let M be a non-negative, non-zero symmetric matrix of order n that is conditionally negative definite. Then M has exactly one positive eigenvalue.*

A general formula for $H_B(G)$ appears in Theorem 3.1 of [1] and was also presented more recently in Theorem 3.1 of [4].

Theorem 2.4. [1, 4] *For a graph G with n vertices and m edges, the Hamming index is*

$$H_B(G) = 2m(n-2).$$

Let μ_1 denote the Hamming spectral radius.

Proposition 2.2. [14] *For a connected graph with $n > 3$ vertices,*

$$n - 1 \leq \mu_1 \leq 2(n-1)(n-2).$$

Equality in the above right-hand inequality occurs if and only if $G \cong K_n$, where K_n is a complete graph on n vertices.

3. Hamming Eigenvalues and Hamming Energy

In this section, we analyze the signs of the Hamming eigenvalues; as a consequence, we establish the general expression for the Hamming energy. Let K_n be the complete graph on n vertices and $\overline{K_n}$ be the complement of K_n.

Theorem 3.1. *Consider a graph G consisting of n vertices and m edges. If $G \notin \{K_2, \overline{K_n}\}$, then the Hamming matrix of G has exactly one positive eigenvalue.*

Proof. If $G \in \{K_2, \overline{K_n}\}$, then the Hamming matrix $H(G)$ is the zero matrix. For all other cases, $H(G)$ is a non-negative and non-zero symmetric matrix. We proceed with the following proof for $H(G)$.

For the vertices v_i and v_j, let $s(v_i) = b_{i1}b_{i2}\cdots b_{im}$ and $s(v_j) = b_{j1}b_{j2}\cdots b_{jm}$. Then the Hamming distance between $s(v_i)$ and $s(v_j)$ is given by

$$H_d(s(v_i), s(v_j)) = \sum_{k=1}^{m} (b_{ik} \oplus_2 b_{jk}).$$

Therefore, the Hamming matrix can be expressed as $H(G) = \sum_{k=1}^{m} H(G)^{[k]}$, in which the matrix $H(G)^{[k]} = (h_{ij}^{[k]})$ is defined by $h_{ij}^{[k]} = b_{ik} \oplus_2 b_{jk}$. For $i = 1, 2, \ldots, n$ and $k = 1, 2, \ldots, m$, let us define the following partition

$$P_k = \{i : b_{ik} = 0\} \text{ and } Q_k = \{i : b_{ik} = 1\}.$$

With this partition, the (i, j)-th entry in the matrix $H(G)^{[k]}$ can be expressed as

$$h_{ij}^{[k]} = \begin{cases} 0, & \text{if both } i, j \in P_k \text{ or both } i, j \in Q_k \\ 1, & \text{otherwise.} \end{cases}$$

Let $y = (y_1, y_2, \ldots, y_n) \in \mathbb{R}^n$. Then

$$y^T H(G)^{[k]} y = \sum_{i,j=1}^{n} y_i y_j h_{ij}^{[k]}.$$

$$y^T H(G)^{[k]} y = \sum_{\substack{i \in P_k \\ j \in Q_k}} y_i y_j + \sum_{\substack{i \in Q_k \\ j \in P_k}} y_i y_j = 2 \left(\sum_{i \in P_k} y_i \right) \left(\sum_{j \in Q_k} y_j \right).$$

If $S_1 = \sum_{i \in P_k} y_i$ and $S_2 = \sum_{j \in Q_k} y_j$, then

$$y^T H(G)^{[k]} y = 2 S_1 S_2.$$

Now, suppose that $\sum_{i=1}^{n} y_i = 0$, then $S_1 + S_2 = 0$, which implies $S_1 = -S_2$. With this, we have

$$y^T H(G)^{[k]} y = 2(-S_2)(S_2) = -2S_2^2 \leq 0.$$

Therefore,

$$y^T H(G)^{[k]} y \leq 0 \quad \text{whenever} \sum_{i=1}^{n} y_i = 0.$$

Hence,

$$y^T H(G) y = \sum_{k=1}^{m} y^T H(G)^{[k]} y \leq 0 \quad \text{whenever} \sum_{i=1}^{n} y_i = 0.$$

This shows that $H(G)$ is a conditionally negative definite matrix. Therefore, by applying Proposition 2.1, we conclude that $H(G)$ has exactly one positive eigenvalue. □

Corollary 3.1. *For any graph G, the Hamming energy of G is $HE(G) = 2\mu_1$.*

Proof. If $G \in \{K_2, \overline{K_n}\}$, then the Hamming energy is zero. In all other cases, the result follows directly from Theorem 3.1 by using the fact that the Hamming matrix has zero trace. □

The following corollary provides a characterization of graphs with equal Hamming energy.

Corollary 3.2. *Let G_1 and G_2 be the graphs of the same order. Then $HE(G_1) = HE(G_2)$ if and only if they have the same Hamming spectral radius.*

Remark 3.1. Although the proof of Theorem 8 of [15] applies the Cauchy–Schwarz inequality incorrectly, the stated lower bound $\frac{2H_B(G)}{n} \leq \mu_1$ remains valid. Indeed, since $H(G)$ is symmetric, the Rayleigh quotient gives

$$\mu_1 \geq \frac{\mathbf{1}^T H(G)\mathbf{1}}{\mathbf{1}^T\mathbf{1}} = \frac{\sum_{i,j} h_{ij}}{n} = \frac{2H_B(G)}{n},$$

where $\mathbf{1}$ denotes the all-one vector.

Theorem 3.2. *Let G be a graph with $n > 4$ vertices and m edges, and let Δ denote its maximum degree. Then*

$$\frac{4m(n-2)}{n} \leq \mu_1 \leq 2m + (n-4)\Delta.$$

Equality in either inequality holds if and only if G is regular.

Proof. Let G be a graph with n vertices and m edges. Denote its vertex set by $\{v_1, v_2, \ldots, v_n\}$, and let $\deg_G(v_i) = k_i$ for $i = 1, 2, \ldots, n$. Let us assume that v_1 is adjacent to $v_2, v_3, \ldots, v_{k_1+1}$ and non-adjacent to $v_{k_1+2}, v_{k_1+3}, \ldots, v_n$. Then the matrix $H(G)$ has the following entries in the row corresponding to v_1:

$$0,\ k_1 + k_2 - 2,\ k_1 + k_3 - 2,\ \ldots,\ k_1 + k_{k_1+1} - 2,\ k_1 + k_{k_1+2},\ k_1 + k_{k_1+3},\ \ldots,\ k_1 + k_n.$$

If the sum of the entries in the i^{th} row of $H(G)$ is denoted by $r_i(H(G))$, then

$$\begin{aligned} r_1(H(G)) &= \sum_{j=2}^{k_1+1} (k_1 + k_j - 2) + \sum_{j=k_1+2}^{n} (k_1 + k_j) \\ &= \sum_{j=2}^{n} k_1 + \sum_{j=2}^{n} k_j - \sum_{j=2}^{k_1+1} 2 \\ &= (n-1)k_1 + (2m - k_1) - 2k_1 \quad \text{as} \sum_{j=1}^{n} k_j = 2m \\ &= 2m + (n-4)k_1. \end{aligned}$$

Similarly, for each $i = 1, 2, \ldots, n$, one can show $r_i(H(G)) = 2m + (n-4)k_i$. Now, by applying Gershgorin's theorem (Theorem 2.3), each eigenvalue μ of $H(G)$ satisfies $|\mu| \leq \max_i r_i(H(G))$ for $i = 1, 2, \ldots, n$. Therefore,

$$\mu_1 \leq \max_i r_i(H(G)) = 2m + (n-4)\Delta.$$

The lower bound can be obtained by substituting the value of $H_B(G)$ from Theorem 2.4 into

$$\frac{2H_B(G)}{n} \leq \mu_1$$

as mentioned in Remark 3.1.

If G is an r-regular graph, then $\Delta = r$ and $m = nr/2$. Substituting these values into the lower and upper bounds yields $2r(n-2)$ in both cases, which is precisely the Hamming spectral radius of G, as stated in [4].

Conversely, we prove that equality in either bound implies that G is regular. If $m = 0$, then G is regular. Hence, assume that $m > 0$.

Suppose first that equality holds in the lower bound. Then

$$\mu_1 = \frac{4m(n-2)}{n} = \frac{\mathbf{1}^T H(G)\mathbf{1}}{\mathbf{1}^T\mathbf{1}}.$$

By the equality condition in the Rayleigh quotient theorem, $H(G)\mathbf{1} = \mu_1\mathbf{1}$. Consequently, for every i,

$$2m + (n-4)k_i = \frac{4m(n-2)}{n}, \text{which gives } (n-4)k_i = \frac{2m(n-4)}{n}.$$

Since $n > 4$, we obtain $k_i = 2m/n$ for every i. Thus, G is regular.

Now, suppose that equality holds in the upper bound. Then

$$\mu_1 = 2m + (n-4)\Delta.$$

Since $n > 4$ and $m > 0$, the matrix $H(G)$ is non-zero, non-negative and irreducible [4]. By the Perron–Frobenius theorem, there exists a positive vector $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$ such that $H(G)x = \mu_1 x$. Since $H(G)$ is symmetric,

$$\mu_1 \sum_{i=1}^{n} x_i = \mathbf{1}^T H(G)x = \sum_{i=1}^{n} r_i(H(G))x_i.$$

Substituting $\mu_1 = 2m + (n-4)\Delta$ and $r_i(H(G)) = 2m + (n-4)k_i$ yields

$$(n-4)\sum_{i=1}^{n}(\Delta - k_i)x_i = 0.$$

Since $n > 4$ and $x_i > 0$, we have $\Delta - k_i = 0$ for every i. Hence, it follows that $k_i = \Delta$ for every i. Therefore, G is regular. □

The following result follows directly from the preceding theorem.

Corollary 3.3. *Let G be a graph with $n > 4$ vertices and m edges, and let Δ denote its maximum degree. Then*

$$\frac{8m(n-2)}{n} \leq HE(G) \leq 2(2m + (n-4)\Delta).$$

Equality in either inequality holds if and only if G is regular.

Theorem 3.3. *Let G be a graph with $n = 4$ vertices and m edges. Then $\mu_1 = 2m$ and $HE(G) = 4m$.*

Proof. As done in the proof of Theorem 3.2, the sum of the entries in the i^{th} row of $H(G)$ is

$$r_i(H(G)) = 2m + (n-4)k_i,$$

where $k_i = \deg_G(v_i)$. Therefore, for $n = 4$, the sum of elements of each row in $H(G)$ is $2m$. Hence

$$H(G)\mathbf{1} = 2m\mathbf{1},$$

where $\mathbf{1}$ denotes the all-one vector. This shows that $2m$ is an eigenvalue of the Hamming matrix $H(G)$. If $G = \overline{K_4}$, then all eigenvalues of $H(G)$ are zero and $m = 0$. Hence $\mu_1 = 0$ and $HE(G) = 0$. If $G \neq \overline{K_4}$, then $2m > 0$. Therefore, by Theorem 3.1, the only positive eigenvalue $H(G)$ is $2m$. Hence, $\mu_1 = 2m$ and $HE(G) = 2\mu_1 = 4m$. □

In [4] it is shown that for an r-regular graph with n vertices, $HE(G) = 4r(n-2)$. Therefore, all r-regular graphs on n vertices have the same Hamming energy. Furthermore, if G is an r-regular graph with n vertices, then its complement graph $\overline{G}$ is a regular graph of degree $n - r - 1$. Therefore, $HE(\overline{G}) = 4(n-r-1)(n-2)$. From this, we observe that for any r-regular graph G with $n \geq 3$ vertices, $HE(G) = HE(\overline{G})$ if and only if $r = (n-1)/2$.

Let $\det(H(G))$ denote the determinant of the matrix $H(G)$.

Corollary 3.4. *Let G be a graph with n vertices and m edges. If $G \notin \{K_2, \overline{K_n}\}$, then $\det(H(G)) \geq 0$ if n is odd, and $\det(H(G)) \leq 0$ if n is even.*

Proof. By Theorem 3.1, $\mu_1 > 0$ and $\mu_i \leq 0$; $i = 2, 3, \ldots, n$. Consequently,

$$\begin{aligned}\det(H(G)) &= \prod_{i=1}^{n} \mu_i = \mu_1 \prod_{i=2}^{n} \mu_i \\ &= (-1)^{n-1} \mu_1 \prod_{i=2}^{n} (-\mu_i).\end{aligned}$$

Since $\mu_1 \prod_{i=2}^{n}(-\mu_i) \geq 0$, the result follows according to whether $n-1$ is even or odd. □

If $H(G)$ is non-singular, then the inequalities in Corollary 3.4 are strict.

Corollary 3.5. *Let G be a graph with $n \geq 2$ vertices and let $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_n$ be the Hamming eigenvalues of G. Then*

$$2(n-1)|\mu_2| \leq HE(G) \leq 2(n-1)|\mu_n|.$$

Proof. If $G \in \{K_2, \overline{K_n}\}$, then all eigenvalues of $H(G)$ are zero. Hence, the result is obvious. Now, suppose that $G \notin \{K_2, \overline{K_n}\}$. Then, by Theorem 3.1, $\mu_1 > 0$ and $\mu_i \leq 0$ for $i = 2, 3, \ldots, n$. Therefore, $\mu_2 \geq \mu_3 \geq \cdots \geq \mu_n$ implies that $|\mu_2| \leq |\mu_3| \leq \cdots \leq |\mu_n|$.

Since the Hamming matrix has trace zero, we have

$$\mu_1 = -\sum_{i=2}^{n} \mu_i = \sum_{i=2}^{n} |\mu_i| \geq (n-1)|\mu_2|$$

and

$$\mu_1 = -\sum_{i=2}^{n} \mu_i = \sum_{i=2}^{n} |\mu_i| \leq (n-1)|\mu_n|.$$

Hence,

$$2(n-1)|\mu_2| \leq 2\mu_1 \leq 2(n-1)|\mu_n|.$$

Consequently, by Corollary 3.1, we get

$$2(n-1)|\mu_2| \leq HE(G) \leq 2(n-1)|\mu_n|.$$

□

The following theorem provides bounds for the Hamming energy of a connected graph in terms of the adjacency energy of K_n.

Theorem 3.4. *For a connected graph G with $n > 3$ vertices,*

$$\mathcal{E}(K_n) \leq HE(G) \leq 2(n-2)\mathcal{E}(K_n).$$

Equality in the above right-hand inequality occurs if and only if $G \cong K_n$.

Proof. The result is obtained directly from Proposition 2.2 and Corollary 3.1 by using the fact that $\mathcal{E}(K_n) = 2(n-1)$. □

4. Conclusion

Vučićević et al. [15] investigated the predictive potential of the Hamming energy with respect to physicochemical properties of octanes. They showed that the Hamming energy of a graph exhibits a stronger correlation with entropy and heat of vaporization than the classical adjacency energy. In the present work, it is shown (see Theorem 3.1) that the graph $G \notin \{K_2, \overline{K_n}\}$ has exactly one positive Hamming eigenvalue and by Corollary 3.1, the Hamming energy is exactly twice the Hamming spectral radius. Therefore, the study of the Hamming energy can be reduced to the study of the Hamming spectral radius.

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References

[1] S. Ali, S. Suwilo, Mardiningsih, The sum of Hamming distance for some composite graphs relative to vertex-edge incidence matrix, *J. Phys. Conf. Ser.* **2421** (2023) #012041.

[2] R. B. Bapat, T. E. S. Raghavan, *Nonnegative Matrices and Applications*, Cambridge University Press, Cambridge, 1997.

[3] B. Borovićanin, N. Stojanović, N. Vučićević, Hamming energy of sunlet and n-barbell graphs, *Sci. Publ. State Univ. Novi Pazar Ser. A Appl. Math. Inform. Mech.* **17** (2025) 15–25.

[4] B. Borovićanin, N. Stojanović, N. Vučićević, On the Hamming spectrum and Hamming energy of graphs, *Discrete Math. Lett.* **17** (2026) 22–29.

[5] D. Cvetković, P. Rowlinson, S. Simić, *An Introduction to the Theory of Graph Spectra*, Cambridge University Press, Cambridge, 2010.

[6] E. N. Gilbert, Gray codes and paths on the n-cube, *Bell System Tech. J.* **37** (1958) 815–826.

[7] I. Gutman, The energy of a graph, *Ber. Math. Stat. Sekt. Forschungszent. Graz.* **103** (1978) 1–22.

[8] I. Gutman, B. Furtula, Graph energies and their applications, *Bull. Cl. Sci. Math. Nat. Sci. Math.* **44** (2019) 29–45.

[9] R. A. Horn, C. R. Johnson, *Matrix Analysis*, Cambridge University Press, Cambridge, 2013.

[10] V. Nikiforov, The energy of graphs and matrices, *J. Math. Anal. Appl.* **326** (2007) 1472–1475.

[11] H. S. Ramane, I. B. Baidari, R. B. Jummannaver, V. V. Manjalapur, G. A. Gudodagi, A. S. Yalnaik, A. S. Hanagawadimath, Hamming index of graphs with respect to its incidence matrix, *Indonesian J. Combin.* **6** (2022) 120–129.

[12] H. S. Ramane, T. Shivaprasad, S. Y. Chowri, On the eigenvalues of Hamming matrix and Hamming energy of a graph, *Creat. Math. Inform.* **34** (2025) 291–298.

[13] I. Redžepović, N. Vučićević, N. Stojanović, On sensitivity of Hamming energy of a graph, *Sci. Publ. State Univ. Novi Pazar Ser. A Appl. Math. Inform. Mech.* **16** (2024) 159–163.

[14] Z. Tang, B. Li, Maximizing the Hamming spectral radius in graphs, *Electron. J. Math.* **11** (2026) 58–69.

[15] N. Vučićević, I. Redžepović, N. Stojanović, Hamming matrix and Hamming energy of a graph, *MATCH Commun. Math. Comput. Chem.* **93** (2025) 713–724.