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Research Article

Nonexistence of Almost Moore Digraphs with Self-Repeats of Diameter Greater Than Two

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Abstract

A (d, k)-digraph is an almost Moore digraph of degree $d > 1$ and diameter $k > 1$, whose order N is one less than the Moore bound, that is, $N = d + d^2 + \cdots + d^k$. Almost Moore digraphs can be characterized as d-regular digraphs for which, for every vertex v, there exists a unique vertex denoted $r(v)$ and called the repeat of v such that there are exactly two directed paths from v to $r(v)$, one of length at most k and the other one of length k. In particular, a self-repeat is a vertex v for which $r(v) = v$. The existence of almost Moore digraphs has only been proven for diameter $k = 2$, and it is believed that there do not exist (d, k)-digraphs for non-trivial values of $k > 2$. In this paper, we prove that there are no (d, k)-digraphs containing self-repeats for any $d > 1$ and $k > 2$.

Keywords: almost Moore digraphs; self-repeats; adjacency matrix; eigenvalues; rank.

2020 Mathematics Subject Classification: 05C35, 05C20, 05C50.

1. Introduction

The study of almost Moore digraphs has received great attention due to the impossibility of the Moore digraphs, proven by Plesnik and Znam [23] in 1974 and independently by Bridges and Toueg [6] in 1980. Almost Moore digraphs are diregular (see [20]) and have defect one, that is, their order is one less than the Moore bound $1 + d + \cdots + d^k$, being $d > 1$ the degree and $k > 1$ the diameter of such digraphs. For a magnificent introduction to them and to the degree/diameter problem, see the masterpiece *Moore graphs and beyond: a survey* written by Miller and Širáň in 2005 [21]. So far, their existence has only been shown for $k = 2$. In this case, Fiol, Alegre and Yebra proved in [12] that the $(d, 2)$-digraphs do exist for any degree. A complete classification was given by Gimbert in [14].

Nowadays, for the remaining cases, it is believed that (d, k)-digraphs do not exist. However, their nonexistence has only been proven in a few cases. Concerning the degree, for $k > 2$, Miller and Friš [19] proved that there are no $(2, k)$-digraphs and Baskoro, Miller, Širáň and Sutton [5] (see also [4]) proved the nonexistence of $(3, k)$-digraphs. For $(4, k)$ and $(5, k)$-digraphs with self-repeats, López, Messegué and Miret [17] proved they do not exist, whereas without self-repeats, Miret, Simanjuntak and Simón [22] have reduced the number of possible structures such digraphs can have. Concerning the diameter, Conde, Gimbert, González, Miret and Moreno [10, 11] showed the nonexistence of $(d, 3)$ and $(d, 4)$-digraphs. Recently, Messegué, Miret and Sillasen [18] proved that (d, k)-digraphs with self-repeats do not exist when the diameter k is large with respect to the degree d, specifically, when $k \geq 2(d-1)^2$. Moreover, there exist two conjectures such that, assuming that either of them is true, the nonexistence of (d, k)-digraphs for any $k > 2$ is proven (see [7, 9, 13]).

It is also worth mentioning that the number of possible structures under the automorphism r of repeats of such digraphs can have was reduced to the degrees $d = 4$ and $d = 5$ (see [8, 22, 24]). Recently, Baskoro, Messegué and Miret [1] extended these results to any degree, showing that in the case the digraph has self-repeats, the remaining vertices have all the same order under r.

In this paper, we introduce a pseudo-digraph associated with an almost Moore digraph, whose vertices are the orbits by the automorphism r of repeats, which we call the *orbital-connection digraph*. From the properties of such pseudo-digraphs, together with the closed relationship with their original digraphs, we prove that almost Moore digraphs with self-repeats do not exist for any diameter $k > 2$.

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2. Almost Moore Digraphs

Almost Moore digraphs have a distinguished automorphism, the so-called automorphism of repeats, denoted by r (see [2]), which assigns the repeat of v to each vertex v. This automorphism, as a permutation on the set of vertices, decomposes into distinct cycles. The so-called permutation cycle structure is a tuple

$$(m_1, m_2, \ldots, m_N), \text{ with } \sum_{j=1}^{N} j m_j = N = d + d^2 + \cdots + d^k,$$

where each value m_j counts the number of these cycles of length exactly j.

Given a (d,k)-digraph G, its adjacency matrix $\boldsymbol{A}$ fulfills the equation

$$\boldsymbol{I} + \boldsymbol{A} + \cdots + \boldsymbol{A}^k = \boldsymbol{J} + \boldsymbol{P}, \tag{1}$$

where $\boldsymbol{J}$ denotes the all-one matrix and $\boldsymbol{P} = (p_{ij})$ is the (0,1)-matrix associated with the permutation r on the set of vertices $V(G)$. Since G has no cycles of length less than k, its adjacency matrix $\boldsymbol{A}$ satisfies

$$\mathrm{Tr}(\boldsymbol{A}^\ell) = 0, \text{ for } \ell = 1, 2, \ldots, k-1, \text{ and } \mathrm{Tr}(\boldsymbol{A}^k) = \mathrm{Tr}(\boldsymbol{P}).$$

Almost Moore Digraphs with Self-Repeats

For $k = 2$, there exists a unique $(d,2)$-digraph with self-repeats for each $d > 1$ (in the case $d = 2$ there exist two more non-isomorphic $(2,2)$-digraphs without self-repeats). Its permutation cycle structure is $(m_1, 0, \ldots)$, that is, with all its vertices self-repeats. More precisely, as proven in [12], such a digraph is the line digraph LK_{d+1} of the complete digraph K_{d+1}. Figure 2.1 shows this digraph for $d = 3$ and $k = 2$, whose order is $3 + 3^2 = 12$, that is, the Kautz digraph with these parameters. In the picture, we have colored in blue the vertices that are at distance 1 from the one in black, and in red the other ones that are at distance two.

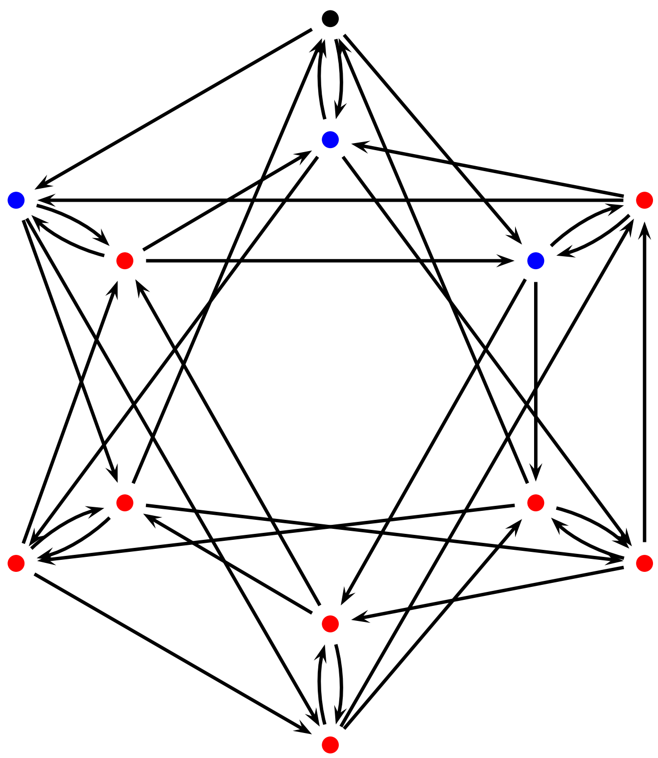

Figure 2.1: Almost Moore digraph of diameter $k = 2$ and degree $d = 3$.

For $k \geq 3$, a (d,k)-digraph G contains either no cycles of length k or exactly one cycle of length k consisting of self-repeats (see [2]). Therefore, when G has self-repeats, it has exactly k self-repeats and

$$\mathrm{Tr}(\boldsymbol{A}^k) = k.$$

According to [1], we have the following result, which will be useful to show the nonexistence of almost Moore digraphs with self-repeats for $k > 2$.

Proposition 2.1. *[1] If there are no (d,k)-digraphs with self-repeats for $k > 2$ with the following permutation cycle structure:*

$$(k, 0, \ldots, 0, m_s, 0, \ldots), \quad N = k + sms, \quad s \mid (d-1), \tag{2}$$

then there does not exist any (d,k)-digraph with self-repeats for $k > 2$.

From now on, taking into account this result, our goal is to show the nonexistence of almost Moore digraphs for $k > 2$ having a permutation cycle structure as in (2).

3. The Pseudo-Digraph $\tilde{G}$ Associated with an Almost Moore Digraph G

In this section, given an almost Moore digraph G, we introduce its pseudo-digraph denoted $\tilde{G}$, which we call the *orbital-connection digraph*. To construct $\tilde{G}$, for each vertex $v \in V(G)$, we consider the orbit $o(v)$ of v by r, that is,

$$o(v) = \left\{ v, r(v), \ldots, r^{ord_r(v)-1}(v) \right\},$$

where $ord_r(v)$ is the order of v with respect to the self-repeat automorphism r. Then, if G is an almost Moore digraph with permutation cycle structure $(m_1, m_2, \ldots, m_N)$, the set of vertices of $\tilde{G}$ is the set of pairwise disjoint orbits of G, that is,

$$V(\tilde{G}) = \{o_1, o_2, \ldots, o_m\}, \text{ with } m = \sum_{j=1}^{N} m_j.$$

Now we define the set of edges of $\tilde{G}$. For each orbit o_i, we choose $v(o_i)$, a vertex in o_i, as a *representative* of o_i. Then we consider an edge from orbits o_i to o_j for every edge connecting $v(o_i)$ with every vertex belonging to o_j. Therefore, there are exactly d directed edges for every vertex of $\tilde{G}$, counting with multiplicities and, whether necessary, loops. Since r is a digraph automorphism, it can be easily seen that the construction of $\tilde{G}$ does not depend on the choice of the representative of each orbit.

We show in Figure 3.1 the digraphs $\tilde{G}$ corresponding to the two non-isomorphic $(2,2)$-digraphs whose permutations of repeats have no self-repeats.

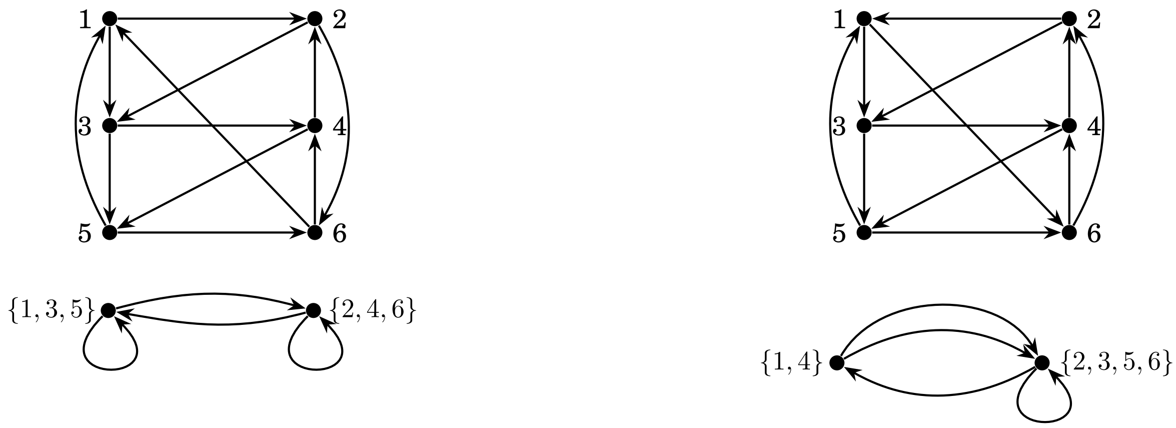

Figure 3.1: Example of digraphs G (top) and its corresponding pseudo-digraph $\tilde{G}$ (bottom) for values $k = d = 2$.

Now, we show some basic properties about the pseudo-digraph $\tilde{G}$.

Lemma 3.1. *For every j, with $1 \leq j \leq m$, it holds:*

i) If $h \neq j$, there is exactly one directed path (in G) of length belonging to the set $\{1, 2, \ldots, k\}$ from $v(o_j)$ to each of the vertices from G defining the orbit o_h.

ii.a) If $|o_j| > 1$, there is exactly one directed path (in G) of length belonging to the set $\{1, 2, \ldots, k\}$ from $v(o_j)$ to each of the vertices from the same orbit excluding the node $v(o_j)$ itself, except exactly for its repeat, for which there are exactly two distinct paths (in G) of length $\leq k$.

ii.b) If $|o_j| = 1$ meaning that $v(o_j)$ is a self-repeat then there is one directed path (inside G) of length belonging to the set $\{1, 2, \ldots, k\}$ from $v(o_j)$ to itself.

Proof. Since r is a digraph automorphism, it is easy to see that this construction does not depend on the choice of the representatives of each orbit. $\square$

Now, using these properties, we can derive a matrix equation that must satisfy the adjacency matrix $\tilde{\boldsymbol{A}}$ of $\tilde{G}$.

Proposition 3.1. *If $\tilde{\boldsymbol{A}}$ is the adjacency matrix of $\tilde{G}$, then*

$$\tilde{\boldsymbol{A}} + \tilde{\boldsymbol{A}}^2 + \cdots + \tilde{\boldsymbol{A}}^k = \boldsymbol{B}, \tag{3}$$

being $\boldsymbol{B}$ the matrix $(b_{ij})_{1\le i,j\le m}$ defined as $b_{ij} = b_j$, where $b_j = |o_j|$.

Proof. The coefficient

$$\left(\tilde{\boldsymbol{A}} + \tilde{\boldsymbol{A}}^2 + \cdots + \tilde{\boldsymbol{A}}^k\right)[j,h]$$

counts the number of paths of length $\in \{1, 2, \ldots, k\}$ in $\tilde{G}$ from vertex o_j to vertex o_h.

This quantity equals the sum of the number of paths of length belonging to the set $\{1, 2, ..., k\}$ in G from vertex $v(o_j)$ to each of the vertices from o_h. If $j \neq h$, by property *i)* of Lemma 3.1, such a number equals $|o_h|$. If $j = h$, by property *ii)* of Lemma 3.1, then if $|o_j| > 1$, such a number is $((|o_h| - 1) - 1) + 2 = |o_h|$; and if $|o_j| = 1$, then such a number is $|o_h| = 1$. $\square$

Characteristic Polynomials of G and $\tilde{G}$

Given a permutation matrix $\boldsymbol{P}$ of order N with permutation cycle structure $(m_1, m_2, \ldots, m_N)$ and the all-one matrix $\boldsymbol{J}$, the characteristic polynomial of $\boldsymbol{J} + \boldsymbol{P}$ is (see [3])

$$\phi(\boldsymbol{J} + \boldsymbol{P}, x) = \det(x\boldsymbol{I} - (\boldsymbol{J} + \boldsymbol{P})) = (x - (N+1))(x-1)^{m_1 - 1}\prod_{i=2}^{N}(x^i - 1)^{m_i}.$$

Its factorization in $\mathbb{Q}[x]$ in terms of cyclotomic polynomials $\Phi_n(x)$ is given by

$$\phi(\boldsymbol{J} + \boldsymbol{P}, x) = (x - (N+1))(x-1)^{m-1}\prod_{n=2}^{N}\Phi_n(x)^{m(n)}, \tag{4}$$

where $m(n) = \sum_{n|i} m_i$ represents the total number of permutation cycles of order a multiple of n, and $m = m(1)$ the total number of permutation cycles, that is, the total number of orbits.

Similarly, the characteristic polynomial of the matrix $\boldsymbol{B} + \boldsymbol{I}$ of order m is given by

$$\phi(\boldsymbol{B} + \boldsymbol{I}, x) = (x - (N+1))(x-1)^{m-1}. \tag{5}$$

Due to the matrix equations (1) and (3) fulfilled by $\boldsymbol{A}$ and $\tilde{\boldsymbol{A}}$, respectively, by substituting x by $1 + x + \cdots + x^k$ in (4) and (5), it turns out that the characteristic polynomials of $\boldsymbol{A}$ and $\tilde{\boldsymbol{A}}$ can be expressed as

$$\begin{aligned}
\phi(\boldsymbol{A}, x) &= (x-d)x^{a_0}\prod_{i\neq 1, i|k}\Phi_i(x)^{a_i}\phi'(\boldsymbol{A}, x), &&\text{with } a_0 + \sum_{i\neq 1, i|k} a_i\varphi(i) = m-1,\\
\phi(\tilde{\boldsymbol{A}}, x) &= (x-d)x^{b_0}\prod_{i\neq 1, i|k}\Phi_i(x)^{b_i}, &&\text{with } b_0 + \sum_{i\neq 1, i|k} b_i\varphi(i) = m-1,
\end{aligned}$$

where $\phi'(\boldsymbol{A}, x)$ is the factor of degree $N - m$ of $\phi(\boldsymbol{A}, x)$ coming from the factor $\prod_{n=2}^{N}\Phi_n(x)^{m(n)}$ of $\phi(\boldsymbol{J} + \boldsymbol{P}, x)$ and $\varphi(i)$ stands for Euler's phi function.

The next result can also be independently proved by using Theorem 9.3.3 from [15].

Proposition 3.2. *The multiplicities a_i and b_i of the factors x and $\Phi_i(x)$ of the characteristic polynomials of $\boldsymbol{A}$ and $\tilde{\boldsymbol{A}}$ satisfy $a_i = b_i$, $\forall i$. As a consequence,*

$$\phi(\tilde{\boldsymbol{A}}, x) \mid \phi(\boldsymbol{A}, x).$$

Proof. Let $V(G) = \{v_1, \ldots, v_j, \ldots, v_N\}$ be the set of vertices of a (d,k)-digraph G with adjacency matrix $\boldsymbol{A}$, and let $\{o_1, \ldots, o_i, \ldots, o_m\}$ be the set of its orbits, that is, the set of vertices of its associated digraph $\tilde{G}$ with adjacency matrix $\tilde{\boldsymbol{A}}$.

If λ is an eigenvalue of $\tilde{\boldsymbol{A}}$ having $\vec{u}$, with $\vec{u}^T = (u_1, \ldots, u_i, \ldots, u_m)$, as an eigenvector of $\tilde{\boldsymbol{A}}$, we define the vector $\vec{w}$, with $\vec{w}^T = (w_1, \ldots, w_j, \ldots, w_N)$, from $\vec{u}$ in the following way:

$$w_j = u_{i_j}, \text{ being } o(v_j) = o_{i_j}.$$

Clearly, this construction is well-defined because each vertex of G belongs to exactly one orbit of $\tilde{G}$. Now, let v_j be any vertex of G and $v_{j_1}, \dots, v_{j_d}$ its neighbours in G. Then the product of the jth row of $\boldsymbol{A}$ times $\vec{w}$ gives

$$w_{j_1} + \cdots + w_{j_d} = u_{i_{j_1}} + \cdots + u_{i_{j_d}}.$$

Since $\vec{u}$ is an eigenvector of $\tilde{\boldsymbol{A}}$ of eigenvalue λ, we get

$$u_{i_{j_1}} + \cdots + u_{i_{j_d}} = \lambda u_{i_j} = \lambda w_j.$$

Therefore, $\vec{w}$ is an eigenvector of $\boldsymbol{A}$ of eigenvalue λ. Furthermore, since $\vec{w}$ is obtained by copying the coordinate values of the vertices at the same orbit, the dimension of the eigenspace of λ in $\tilde{G}$, is at most the dimension of the eigenspace of the same scalar λ in G. In other words, $b_i \leq a_i$ for any i. Since

$$a_0 + \sum_{i \neq 1, i|k} a_i \varphi(i) = b_0 + \sum_{i \neq 1, i|k} b_i \varphi(i) = m - 1,$$

together with $a_i, b_i \geq 0$ it turns out that $a_i = b_i, \forall i \geq 0$. Then the claim follows. $\square$

Bounds of the Multiplicities of the Irreducible Factors of $\phi(\tilde{\boldsymbol{A}}, x)$

We study the multiplicities of the factors of the characteristic polynomial of $\tilde{A}$

$$\phi(\tilde{\boldsymbol{A}}, x) = (x - d)x^{a_0} \prod_{i \neq 1, i|k} \Phi_i(x)^{a_i}, \text{ with } a_0 + \sum_{i \neq 1, i|k} a_i \varphi(i) = m - 1. \tag{6}$$

First, we present a result that is a generalization of a result given by Hefner in [16].

Lemma 3.2. *Let $\boldsymbol{C}$ be an $m \times m$ matrix of non-negative integers such that the sum of the entries of each row equals d and at every column there is at least one non-zero entry. Then*

$$\textit{rank}(\boldsymbol{C}) \geq m/d.$$

Proof. Let us suppose that $\vec{c}, \vec{c}_1, \dots, \vec{c}_k$ are rows from $\boldsymbol{C}$ and $\lambda_1, \dots, \lambda_k$ non-zero scalars such that

$$\vec{c} = \lambda_1 \vec{c}_1 + \cdots + \lambda_k \vec{c}_k. \tag{7}$$

If there is a non-zero entry at position j from $\vec{c}$, then there must exist at least one index i with $1 \leq i \leq k$ such that the entry at position j from $\vec{c}_i$ is non-zero, otherwise Equation (7) cannot hold.

Now, set $g = \text{rank}(\boldsymbol{C})$ and suppose without loss of generality that $\vec{c}_1, \dots, \vec{c}_g$ are g linearly independent rows from $\boldsymbol{C}$. Define then $\boldsymbol{C}'$ to be the $g \times m$ matrix associated with the rows $\vec{c}_1, \dots, \vec{c}_g$.

We showed that every column from $\boldsymbol{C}'$ contains at least one non-zero entry. So, if S is the sum of the entries from matrix $\boldsymbol{C}'$, then $S \geq m$. Furthermore, since the sum of the entries from every row is d and there are exactly g rows, it turns out that $S = d \cdot g$. From here, we obtain that $\text{rank}(\boldsymbol{C}) = g \geq m/d$. $\square$

Proposition 3.3. *The multiplicity a_0 of the factor x in $\phi(\tilde{\boldsymbol{A}}, x)$ fulfills*

$$a_0 \leq m\left(1 - \frac{1}{d}\right).$$

Proof. Since the adjacency matrix $\tilde{\boldsymbol{A}}$ of the pseudo-digraph $\tilde{G}$ associated with a (d,k)-digraph with m orbits satisfies the assumptions of Lemma 3.2 and taking into account that $m = \dim\ker\tilde{\boldsymbol{A}} + \text{rank}(\tilde{\boldsymbol{A}})$, it turns out that

$$a_0 = \dim\ker\tilde{\boldsymbol{A}} \leq m\left(1 - \frac{1}{d}\right).$$

$\square$

Proposition 3.4. *The multiplicities a_i of the factors $\Phi_i(x)$ in $\phi(\tilde{\boldsymbol{A}}, x)$ fulfill*

$$\sum_{i \neq 1, i|k} a_i \varphi(i) \leq 2d + d^2 + \cdots + d^{k-2}.$$

Proof. From the factorization of $\phi(\tilde{A}, x)$ in $\mathbb{Q}[x]$ given in (6) it turns out

$$\mathrm{Tr}(\tilde{\boldsymbol{A}}^{\ell}) = d^{\ell} + \mathcal{S}_{\ell}, \text{ with } \mathcal{S}_{\ell} = \sum_{i \neq 1, i|k} a_i S_{\ell}(\Phi_i(x)), \tag{8}$$

where $S_{\ell}(a(x))$ denotes the sum of the ℓth powers of all roots of $a(x)$.

It is known that these sums can be expressed in terms of the Möbius's function

$$S_{\ell}(\Phi_n(x)) = \sum_{j|\gcd(n,\ell)} \mu\left(\frac{n}{j}\right) j.$$

In particular, since $\gcd(k-1, k) = 1$, we get

$$\mathcal{S}_{k-1} = \mathcal{S}_1. \tag{9}$$

Besides, the roots of $\prod_{i \neq 1, i|k} \Phi_i(x)^{a_i}$ are roots of unity of order a divisor of k. Hence

$$\mathcal{S}_k = \sum_{i \neq 1, i|k} a_i \varphi(i). \tag{10}$$

From the matrix equation (3), we have that

$$\mathrm{Tr}(\tilde{\boldsymbol{A}}) + \mathrm{Tr}(\tilde{\boldsymbol{A}}^2) + \cdots + \mathrm{Tr}(\tilde{\boldsymbol{A}}^k) = \mathrm{Tr}(\boldsymbol{B}) = d + d^2 + \cdots + d^k.$$

By substituting the values of $\mathrm{Tr}(\tilde{\boldsymbol{A}}^{\ell})$ given in (8), it turns out

$$\mathcal{S}_1 + \mathcal{S}_2 + \cdots + \mathcal{S}_{k-1} + \mathcal{S}_k = 0.$$

Taking into account (9) and (10), we have that

$$\sum_{i \neq 1, i|k} a_i \varphi(i) = -(2\mathcal{S}_1 + \mathcal{S}_2 + \cdots + \mathcal{S}_{k-2}).$$

Since $\mathrm{Tr}(\tilde{\boldsymbol{A}}^{\ell}) \geq 0$, from (8), we get $-\mathcal{S}_{\ell} \leq d^{\ell}$, for $1 \leq \ell \leq k-2$. Therefore, $\sum_{i \neq 1, i|k} a_i \varphi(i) \leq 2d + d^2 + \cdots + d^{k-2}$. $\square$

4. Nonexistence of (d, k)-Digraphs with Self-Repeats and $k > 2$

Finally, in this section, we prove the nonexistence of (d, k)-digraphs with self-repeats when $k > 2$ using the results obtained on their associated pseudo-digraphs.

Theorem 4.1. *There do not exist (d, k)-digraphs with self-repeats for $k > 2$.*

Proof. From Proposition 2.1 in order to prove their nonexistence, it is enough to prove the nonexistence of (d, k)-digraphs having this particular permutation structure

$$(k, 0, \ldots, 0, m_s, 0, \ldots), \text{ with } k + s m_s = N = d + d^2 + \cdots + d^k, \text{ and } s \mid (d-1).$$

Thus, let G be a (d, k)-digraph with such a permutation structure. Its associated pseudo-digraph $\tilde{G}$ has order $m = k + m_s$. From the factorization of its characteristic polynomial, we have that

$$m = 1 + a_0 + \sum_{i \neq 1, i|k} a_i \varphi(i).$$

Considering Propositions 3.3 and 3.4, it turns out that $m \leq 1 + m - \dfrac{m}{d} + 2d + d^2 + \cdots + d^{k-2}$, that is,

$$m \leq d + 2d^2 + d^3 + \cdots + d^{k-1}.$$

In particular, we also have

$$m_s = m - k \leq d + 2d^2 + d^3 + \cdots + d^{k-1}.$$

Then, taking into account that $s \le d-1$, we get, when $k \ge 4$,

$$N = k + sm_s \le s(k + m_s) = sm \le (d-1)\left(d + 2d^2 + d^3 + \ldots + d^{k-1}\right) = -d - d^2 + d^3 + d^k < d + \ldots + d^k = N,$$

which is a contradiction. The conclusion follows because the case $k = 3$ cannot hold as shown in [10]. □

Taking into account the result proved in Theorem 4.1 about the nonexistence of (d, k)-digraphs with self-repeats for $k > 2$, together with the study of their permutation cycle structures given in [1], we can conclude that it is enough to show that there do not exist (d, k)-digraphs, $k > 2$, with structures

$$(0, \ldots, 0, m_s, 0, \ldots), \text{ with } N = sms,$$
$$(0, \ldots, 0, m_s, 0, \ldots, 0, m_{2s}, 0, \ldots), \text{ with } N = sms + 2sm_{2s},$$

in order to prove the nonexistence of almost Moore digraphs for diameter greater than 2.

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