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Research Article

On Restrained Coalitions in Graphs: Bounds and Exact Values

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Abstract

A subset $D \subseteq V$ is a dominating set of a graph G with vertex set V if every vertex $v \in V \setminus D$ is adjacent to a vertex in D. Two subsets of V form a coalition if neither of them is a dominating set, but their union is a dominating set. A coalition partition of G is its vertex partition π such that every non-dominating set of π is a member of some coalition, and every dominating set is a single-vertex set in π. The coalition number $C(G)$ of a graph G is the maximum cardinality of its coalition partitions. A subset $R \subseteq V$ is a restrained dominating set if R is a dominating set and any vertex of $V \setminus R$ has at least one neighbor in $V \setminus R$. Restrained dominating coalition, restrained dominating partition and restrained coalition number $RC(G)$ are defined by the same way. In this paper, we prove that $RC(G) \leq C(G)$ for an arbitrary graph G. In addition, some previous results from [A. H. S. Nesam, S. Amutha, N. Anbazhagan, *Stat. Optim. Inf. Comput.* **14** (2025) 3409–3417] are corrected by determining the exact value of the restrained coalition number of cycles.

Keywords: restrained dominating set; coalition partition; coalition number; coalition graph.

2020 Mathematics Subject Classification: 05C69.

1. Introduction

In the present paper, finite simple connected graphs $G(V, E)$ are considered. The order of a graph is the number of its vertices, $n = |V(G)|$. The minimum and maximum vertex degree of a graph G is denoted by $\delta(G)$ and $\Delta(G)$, respectively. The open neighborhood $N(v)$ of a vertex v in G is the set of neighbors of v, while the closed neighborhood of v is $N[v] = N(v) \cup \{v\}$. The path, the cycle, the star, and the complete graph of order n are denoted by P_n, C_n, S_n, and K_n, respectively.

A subset $D \subseteq V(G)$ is a dominating set if every vertex $v \in V(G) \setminus D$ is adjacent to a vertex of D. The domination theory is a classical branch in the graph theory that has found numerous applications. For a broader introduction to domination in graphs, we refer the reader to books [13, 16, 21–23, 25]. A dominating set D is called a restrained dominating set (RD-set) if every vertex $v \in V(G) \setminus D$ is adjacent to at least one vertex of $V(G) \setminus D$, that is, the induced subgraph on vertices $V(G) \setminus D$ has no isolated vertex. The restrained domination number $\gamma_r(G)$ is the minimum cardinality of RD-sets in G. The concept of restrained domination was introduced in [12]. The main results on restrained domination up to 2020 were collected in [16]. So far, hundreds works have been devoted to this topic (see selected articles [5, 7, 8, 27]).

In [17], Haynes et al. introduced the concept of coalitions in graphs. Two subsets of $V(G)$ form a dominating coalition if neither of them is dominating, but their union is a dominating set. A coalition partition of G is its vertex partition π such that every non-dominating set of π is a member of some coalition, and every dominating set is a single-vertex set in π. The coalition number $C(G)$ of a graph G is the maximum cardinality of its coalition partitions. While coalition in graphs is derived from domination in graphs [3, 10, 18], numerous variants of coalition associated with other types of domination have been explored. For example, connected coalition [2, 9], total coalition [1, 4, 24], double total coalition [6, 15] and total restrained coalition [7] correspond to connected, total, double total and total restrained domination in graphs, respectively. One of the recent variations is the restrained dominating coalition [26]. Let V_1 and V_2 be two disjoint non-RD-sets of $V(G)$. They form a restrained dominating coalition (RD-coalition) if their union is an RD-set. A vertex partition $\pi = \{V_1, V_2, \ldots, V_k\}$ of $V(G)$ is called a restrained dominating coalition partition (RD-partition) of a graph G if every non-RD-set of π is a member of an RD-coalition, and every RD-set in π is a single-vertex set. The restrained dominating coalition number $RC(G)$ of a graph G is the maximum cardinality of RD-partitions of G. The restrained dominating coalition graph, denoted by $RCG(G, \pi)$, is obtained by associating the partition sets of π with the vertices of the graph, and two vertices are adjacent if and only if the corresponding sets form an RD-coalition in G. The characterization of coalition graphs generated by coalition partitions of paths, cycles, trees, and subcubic graphs have been studied for domination, total domination, and connected domination in [9–11, 14, 19, 20].

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In this paper, we determine the restrained dominating coalition number for some classes of graphs. In particular, we show that $RC(G) \le C(G)$ for an arbitrary graph G. We shall correct the restrained dominating coalition number of cycles which was obtained in [26].

2. Upper Bounds on the Restrained Coalition Number

We show that every RD-partition of a graph G can be transformed into a coalition partition of G with larger or equal cardinality. This implies that a restrained dominating coalition number cannot exceed the coalition number.

Theorem 2.1. *Let G be a graph of order $n \ge 2$. Then $RC(G) \le C(G)$.*

Proof. Let $\pi = \{V_1, V_2, \dots, V_k\}$ be an RD-partition of cardinality $k = RC(G)$ and let π' be a dominating partition of cardinality $k' \ge k$ obtained from π.

1. If $G = K_n$, then $RC(G) = C(G) = n$. Further, we assume that G is not the complete graph.

2. If each member of π is not a dominating set, then $\pi' = \pi$.

3. Let us show that if the partition π contains two dominating sets that are not RD-sets, then $k = 2$. Let V_1 and V_2 be such sets. Then there is a vertex $v \notin V_1$ for which $N(v) \subseteq V_1$ and a vertex $u \notin V_2$ for which $N(u) \subseteq V_2$. Since V_1 is a dominating set, $V_1 \cap N[u] \neq \emptyset$. It is possible only if $u \in V_1$. By the similar reasoning, $v \in V_2$. Therefore, $\pi = \{V_1, V_2\}$ and V_1 forms an RD-coalition with V_2. Indeed, it is easy to see that the union of $X \in \pi \setminus \{V_1, V_2\}$ with any set of π is not an RD-set. Hence, $k = 2$. Since $C(G) \ge 2$ for any graph, $RC(G) \le C(G)$.

4. Let the partition π contains the unique dominating set V_1 that is not an RD-set. Then there is a vertex $v \notin V_1$ such that $N(v) \subseteq V_1$. Suppose that $V_1 \cup V_2$ is an RD-coalition. It implies that $v \in V_2$, else $N(v) \subseteq V_1 \cup V_2$ and, therefore, the union $V_1 \cup V_2$ is not an RD-coalition. Let $X, Y \in \pi \setminus \{V_1, V_2\}$. Since $N(v) \subseteq V_1 \cup X$, the union $V_1 \cup X$ is not an RD-set. The vertex v is not dominated by $X \cup Y$ for any Y. Therefore, X must form an RD-coalition only with V_2.

4.1. Let the set V_1 has no vertex of degree $n-1$. Then V_1 can be presented as the union of two disjoint subsets $V_1 = M \cup \{w_1, w_2, \dots, w_m\}$, $m \ge 1$, where M is a maximal non-dominating set. Consider partition $\pi' = \{\{w_1\}, \{w_2\}, \dots, \{w_m\}, M, V_2, V_3, \dots, V_k\}$ of cardinality $k + m$. By construction, every member of π' is a non-dominating set, and $M \cup \{w_i\}$ is a dominating set for all $i = 1, 2, \dots, m$ as well as $V_2 \cup V_j$ for all $j = 3, 4, \dots, k$. Hence, $C(G) \ge RC(G)$.

4.2. Let the set V_1 contains vertices $u_1, u_2, \dots, u_s$ of degree $n-1$, and the set $W = V_1 \setminus \{u_1, u_2, \dots, u_s\}$ has m vertices of degree less than $n-1$, where $s \ge 1$ and $m \ge 0$.

If W is not a dominating set and $W \cup V_2$ is a dominating set, then $\pi' = \{\{u_1\}, \{u_2\}, \dots, \{u_s\}, W, V_2, V_3 \dots, V_k\}$ is a desired partition of cardinality $k + s > k$. Hence, $C(G) > RC(G)$.

If $W \cup V_2$ is not a dominating set, then $W \cup V_2 \cup V_j$ is a dominating set for all $j \ge 3$. In this case, we have $\pi' = \{\{u_1\}, \{u_2\}, \dots, \{u_s\}, W \cup V_2, V_3, \dots, V_k\}$ of cardinality $s + k - 1 \ge k$. Consequently, $C(G) \ge RC(G)$.

If W is a dominating set, then consider partition $\{\{u_1\}, \{u_2\}, \dots, \{u_s\}, W, V_2, V_3 \dots, V_k\}$ of cardinality $k + s > k$. To obtain partition π', we further apply the above splitting procedure to decompose W into a maximal non-dominating subset and several single-vertex sets in π'. Therefore, $C(G) > RC(G)$. □

From the proof of Theorem 2.1, an RD-partition π of a graph G has at most two dominating sets that are not RD-sets. If π contains two such sets, then $RCG(G)$ is K_2. An example of such a graph is given in Figure 2.1 (left graph). The sets V_1 and V_2 of the RD-partition consist of black and white vertices, respectively. If π has one dominating set that is not an RD-set, then the coalition graph is a star graph. The right graph in Figure 2.1 has RD-partition $\pi = \{V_1, V_2, V_3, V_4\}$. The set V_1 is a dominating set that is not an RD-set. The restrained coalition graph is S_4.

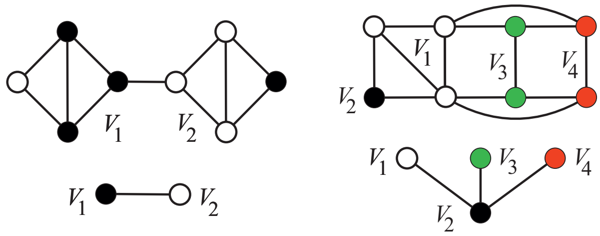

Figure 2.1: Graphs G having dominating sets that are not RD-sets and their coalition graphs (each V_i consists of all vertices of the same color).

By Theorem 2.1, all upper bounds for $C(G)$ are also valid for $RC(G)$. It is known that $C(G) \leq (\Delta(G)+3)^2/4$ for any graph G [18]. There are graphs G with $\Delta(G) = 3$ (and $\delta(G) = 2$) for which $RC(G) = (\Delta+3)^2/4$. For example, a subcubic graph in Figure 2.2 has $RC(G) = 9$. Every black vertex of G is a singleton set. An infinite family of subcubic graphs H having $RC(H) = 9$ can be constructed from G as shown in Figure 2.2.

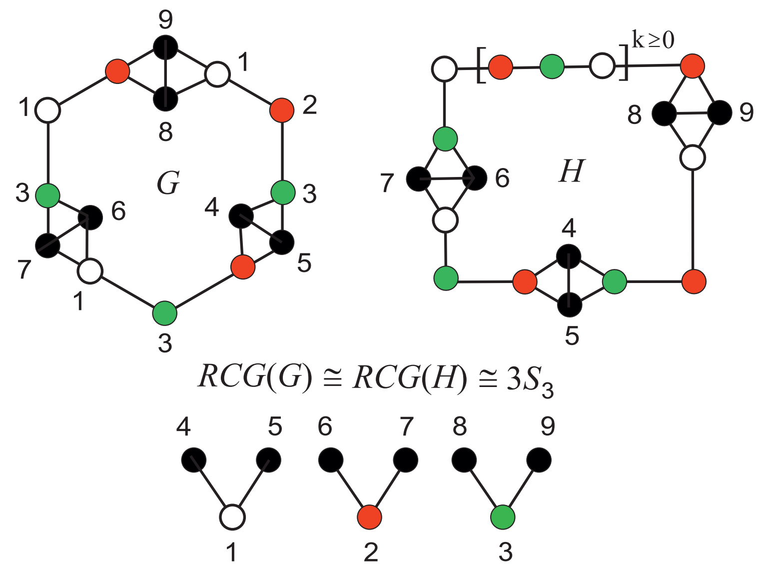

Figure 2.2: Subcubic graphs G and H having $RC(G) = RC(H) = 9$ (each partition set consists of all vertices of the same color, with the exception of black vertices, which are single-vertex sets).

What is the difference between the coalition number and the restrained coalition number? In Table 2.1, the number of connected graphs of order up to 9 vertices with a difference between $C(G)$ and $RC(G)$ is displayed.

Table 2.1: Difference $d = C(G) - RC(G)$ for graphs of order n.

$n \setminus d$	0	1	2	3	4	5	6	graphs
6	77	25	9	1	.	.	.	. 112
7	580	226	43	3	1	.	.	853
8	8183	2399	511	21	2	1	.	11117
9	209769	41717	9169	396	26	2	1	261080

Proposition 2.1. *For every $k \geq 0$, there exists a graph G with $C(G) - RC(G) = k$.*

Proof. Consider graph G of order n shown in Figure 2.3. It is easy to verify that $C(G) = n$. The sets V_1, V_2, V_3 consist of black vertices, one red vertex, and white vertices, respectively. The sets V_2 and V_3 are not dominating, and V_1 is a dominating set that is not an RD-set. The restrained coalition graph of G is S_3. Now we prove that $RC(G) = 3$. Let $\pi = \{V_1, V_2, V_3, \dots\}$ be an RD-partition of G. If vertices u and v belong to the same RD-coalition, then this coalition should also contain all vertices of degree 2 (see the right graph in Figure 2.3). Then π consists of at most three sets. Suppose $u \in V_1$, $v \in V_2$ and $V_1 \cup V_2$ is not an RD-coalition. Let V_3 be a coalition partner for V_1. Since w is a pendant vertex, $w \in V_1 \cup V_3$. If $w \in V_1$, then there is no coalition partner for V_2 in π. Hence, $w \in V_3$. Let π includes a set V_4 which has only vertices of degree 2. We show that there is no RD-coalition for V_4. Since every coalition must contain the pendant vertex w, the set V_4 can form an RD-coalition only with V_3. If the union of V_4 and V_3 does not contain a vertex x of degree 2, then $V_4 \cup V_3$ does not dominate x. If all vertices of degree 2 are in $V_4 \cup V_3$, then all neighbors of the vertex u belong to the dominating set. As a result, we obtain $C(G) - RC(G) = n - 3$. □

In the next result, we establish an upper bound on the restrained coalition number in terms of the restrained domination number $\gamma_r(G)$.

Proposition 2.2. *Let G be a graph of order $n \geq 2$ and $\gamma_r(G) \geq 2$. Then*

$$RC(G) \leq n - \gamma_r(G) + 2$$

and the bound is sharp for complete bipartite graph $K_{r,s}$, $r \geq s \geq 2$.

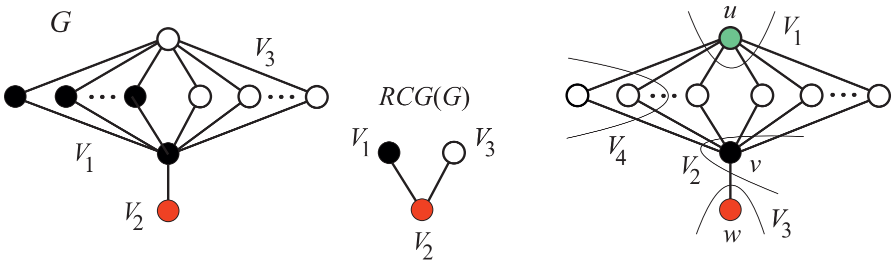

Figure 2.3: Graph G of order n having $C(G) = n$ and $RC(G) = 3$ (each V_i consists of all vertices of the same color).

Proof. Let $\pi(G) = \{V_1, V_2, \ldots, V_k\}$ be an RD-partition and $k = RC(G)$. Since $\gamma_r(G) \geq 2$, the partition does not contain a singleton RD-set. Assume that V_1 and V_2 form an RD-coalition. Then $V_1 \cup V_2$ is an RD-set of G, and thus

$$\gamma_r(G) \leq |V_1| + |V_2|.$$

Accordingly,

$$n = |V_1| + |V_2| + |V_3| + \cdots + |V_k| \geq |V_1| + |V_2| + (k-2) \geq \gamma_r(G) + (k-2)$$

leading to the desired upper bound. □

A similar bound is also valid for the dominating coalition number based on different types of domination in graphs.

3. Restrained Coalition Partitions of Trees

It is known the following upper bound for graphs with pendant vertices.

Proposition 3.1. [26] *Let G be a graph with $\delta(G) = 1$. Then $RC(G) \leq \Delta(G) + 2$.*

The set of restrained coalition graphs of paths contains only two graphs.

Proposition 3.2. [26] *Path P_n of order n has coalition graphs S_2 for $2 \leq n \leq 5$ and S_3 for $n \geq 6$.*

It is easy to see that all pendant vertices of a tree T must belong to every RD-set. This immediately implies that if $RC(T) \geq 3$, then all pendant vertices belong to one set of an RD-partition, and every RD-coalition includes this set. Therefore, $RCG(T)$ of a tree T is always a star graph.

Proposition 3.3. *For every $\Delta \geq 3$ and every $n \geq 4\Delta + 1$, there exist trees T of order n with the maximum degree $\Delta(T) = \Delta$ such that the restrained coalition graph of T is $S_{\Delta+2}$.*

Proof. Consider a tree T with $\Delta(T) \geq 3$ and its RD-partition shown in Figure 3.1. By increasing the order of the bottom path with white vertices, we have a tree of an arbitrary order $n \geq 4\Delta + 1$ with given Δ. The corresponding coalition graph is the star graph $S_{\Delta+2}$. □

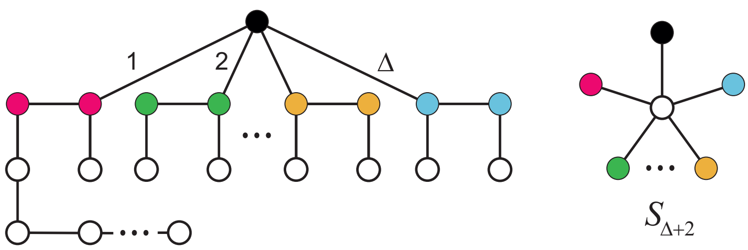

Figure 3.1: A tree and its coalition graph (each partition set consists of all vertices of the same color).

The numbers of trees with given restrained coalition number are presented in Table 3.1.

Table 3.1: Number of trees of order n with given $RC(T)$.

$RC(T) \setminus n$	4	5	6	7	8	9	10	11	12	13
2	2	3	5	8	13	20	34	54	95	160
3	.	.	1	3	10	26	67	155	358	792
4	.	.	.	.	.	1	5	26	98	348
5	.	.	.	.	.	.	.	.	.	1
total	2	3	6	11	23	47	106	235	551	1301

The existence of cycle-containing graphs with arbitrary value $C(T) - RC(T)$ is established in Proposition 2.1. The following result shows that this property is also valid for trees.

Proposition 3.4. *For every $k \geq 0$, there exists a tree T with $C(T) - RC(T) = k$.*

Proof. For $k = 0, 1, 2$, we have $C(P_2) - RC(P_2) = 2 - 2 = 0$, $C(P_3) - RC(P_3) = 3 - 2 = 1$, and $C(P_6) - RC(P_6) = 5 - 3 = 2$ (see [17,26]). Consider tree T of order $n \geq 7$ having $\Delta(T) = \deg(v) \geq 3$ depicted in Figure 3.2. We show that $C(T) = \Delta(T) + 2$. Suppose that $C(T) > \Delta(T) + 2 \geq 5$. Then vertices of degree 1 and 2 are contained in at least $\Delta(T) + 2$ different sets of the coalition partition. Thus, there exist two pendant vertices x and y such that the vertices of $N[x] \cup N[y]$ are contained in four different sets of the coalition partition. Since every coalition must contain a vertex of $N[x]$ and a vertex of $N[y]$, we have $C(T) \leq 4$, a contradiction. The following vertex partition of T has cardinality $\Delta(T) + 2$: vertex v and every vertex of degree 2 are singleton sets, and all pendant vertices form one set.

Assume that V_1, V_2 are sets of an RD-partition π of T such that $v \in V_1$ and $V_1 \cup V_2$ form an RD-coalition. Since the set of all pendant vertices X_e of a graph must belong to every RD-coalition, $X_e \subset V_1 \cup V_2$. If there exists $V_3 \in \pi$ and V_3 contains a vertex of degree 2, then $V_1 \cup V_2$ is not an RD-coalition (all neighbors of this vertex will be in the dominating set). Hence, the set $V_1 \cup V_2$ includes all vertices of T, and we get $RC(T) = 2$. Then $C(T) - RC(T) = \Delta(T) \geq 3$. □

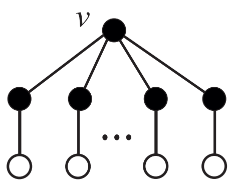

Figure 3.2: Tree T having $C(T) - RC(T) = \Delta(T)$. The domination coalition partition is composed of a set of white vertices and singleton sets containing black vertices. The RD partition is formed by two sets of vertices with the same color.

Table 3.2 shows the numbers of trees T of order up to 13 with the given difference of the coalition numbers.

Table 3.2: Difference $d = C(T) - RC(T)$ for trees of order n.

$d \setminus n$	3	4	5	6	7	8	9	10	11	12	13
0	0	0	0	0	0	0	1	3	13	36	105
1	1	1	1	1	2	5	11	26	62	160	421
2	.	1	2	5	8	17	32	70	143	305	635
3	.	.	.	.	1	1	2	6	14	45	126
4	.	.	.	.	.	.	1	1	2	4	11
5	.	.	.	.	.	.	.	.	1	1	2
6	.	.	.	.	.	.	.	.	.	.	1
trees	1	2	3	6	11	23	47	106	235	551	1301

4. Restrained Coalition Number of Cycles

It is known that $C(C_n) \leq 6$ (see [17]). By Theorem 2.1, we have $RC(C_n) \leq 6$. The restrained coalition number of cycles was previously reported in [26].

Proposition 4.1. [26] *For the cycle C_n,*

$$RC(C_n) = \begin{cases} n, & \text{if } n = 3, 4 \\ 4, & \text{if } n \equiv 0 \mod 4 \\ 6, & \text{if } n \equiv 0 \mod 3 \\ 3, & \text{otherwise.} \end{cases}$$

In what follows, we formulate a correct version of Proposition 4.1.

Proposition 4.2. *For the cycle C_n,*

$$RC(C_n) = \begin{cases} 3, & \text{if } n = 3, 5 \\ 4, & \text{if } n = 4, 8 \\ 5, & \text{if } n = 7, 11 \\ 6, & \text{otherwise.} \end{cases}$$

Proof. It is easy to verify that $RC(C_3) = RC(C_5) = 3$, $RC(C_4) = 4$, and $RC(C_6) = 6$. The restrained coalition numbers of cycles of order $n = 7, 11$ can be found by computer (generation and checking all RD-partitions). Examples of suitable RD-partitions and the corresponding restrained coalition graphs are presented in Figure 4.1. For cycles with $RC(C_n) = 6$, RD-partitions are shown in Figure 4.2 ($n = 9, 12, 15$) and Figure 4.3 (other cycles). The order of cycles in Figure 4.3 is equal to $n = 3m + 4k + 7$, where $m \geq 1$ and $k \geq 0$. It is not hard to check that n goes over all values $10, 13, 14, 16, 17, 18, \ldots$. □

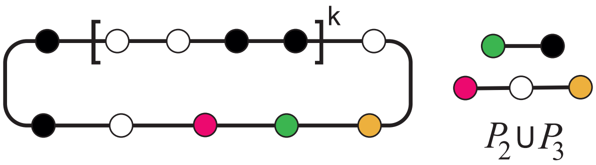

Figure 4.1: Cycles C_7 and C_{11} having restrained coalition number 5, $k = 0, 1$ (each partition set consists of all vertices of the same color).

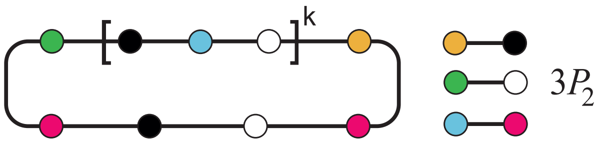

Figure 4.2: Cycles C_9, C_{12} and C_{15} having restrained coalition number 6, $k = 1, 2, 3$ (each partition set consists of all vertices of the same color).

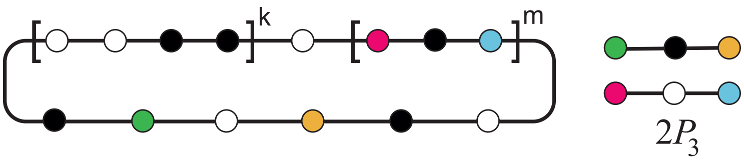

Figure 4.3: Cycles C_n of order $n = 3m + 4k + 7$ having $RC(C_n) = 6$, $m \geq 1$, $k \geq 0$ (each partition set consists of all vertices of the same color).

5. Conclusion

In this paper, we have studied properties of coalition partitions based on the restrained domination in graphs. An upper bound on the RD-coalition number of a graph is established in terms of its coalition number. Infinite families of graphs having the maximal RD-coalition number are constructed. It is shown that the difference between the RD-coalition number and the coalition number can be arbitrary large. The restrained coalition numbers of cycles and trees are also studied, which corrects some previous results.

We propose the following open problems:

1. Characterize all graphs G satisfying $RC(G) = C(G)$.

2. Characterize coalition graphs generated by RD-partitions of cycles.

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