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Research Article

Improved bounds on the domatic numbers of queens graphs

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Abstract

The queens graph Q_n, for any positive integer n, has the n^2 squares of the $n \times n$ chessboard as its vertices with two vertices adjacent if and only if they are on the same row, column, or diagonal of the board. Recently, Hedetniemi et al. [*Congr. Numer.* **235** (2024) 5–21] determined exact values or provided bounds for the domatic, idomatic, and total domatic number of Q_n for n up to 9. For an undirected graph G, these numbers are denoted by $\mathrm{dom}(G)$, $\mathrm{idom}(G)$, and $\mathrm{tdom}(G)$, respectively, and defined as the maximum order k for which a partition $\{V_1, V_2, \ldots, V_k\}$ of $V(G)$ into (dominating, independent dominating, total dominating) sets exists. In this paper, using a computational approach based on SAT solvers, some of their open questions are settled precisely, and improvements upon their bounds are obtained. In particular, for the domatic number, it is shown that $\mathrm{dom}(Q_7) = 11$, $\mathrm{dom}(Q_9) = 13$, $\mathrm{dom}(Q_{10}) = 16$, $\mathrm{dom}(Q_{11}) \in \{18, 19, 20\}$, and $\mathrm{dom}(Q_{12}) \in \{20, 21\}$. For the idomatic number, it is shown that $\mathrm{idom}(Q_9) = 14$, $\mathrm{idom}(Q_{10}) \in \{15, 16\}$, $\mathrm{idom}(Q_{11}) = 17$, and $\mathrm{idom}(Q_{12}) \in \{18, 19, 20\}$. Finally, for the total domatic number, it is established that $\mathrm{tdom}(Q_9) = 14$, $\mathrm{tdom}(Q_{10}) \in \{15, 16\}$, $\mathrm{tdom}(Q_{11}) \in \{16, 17\}$, and $\mathrm{tdom}(Q_{12}) \in \{18, 19, 20\}$.

Keywords: graphs; domination; domatic number; computational combinatorics.

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1. Introduction

Let $G = (V, E)$ be a simple undirected graph with vertex set V and edge set E. To reduce clutter, we write an element $\{u, v\} \in E$ as uv. Here, the two vertices u and v are *adjacent* whenever $uv \in E$. The *open neighborhood* of a vertex $v \in V$, denoted by $N(v)$, is the set of neighbors of v excluding v itself, i.e., $N(v) = \{u | uv \in E\}$. Similarly, the *closed neighborhood* of v, denoted by $N[v]$, is the set of neighbors of v including v, i.e., $N[v] = \{v\} \cup N(v)$.

Let S be a subset of V. We say that S is an *independent set* if no two vertices in S are adjacent. On the other hand, S is a *dominating set* if every vertex in $V - S$ is adjacent to at least one vertex in S. The *domination number* of a graph G, denoted by $\gamma(G)$, is equal to the minimum cardinality of a dominating set in G. If S is both an independent set and a dominating set, we say that S is an *independent dominating set*. Then, the *independent domination number* of a graph G, denoted by $i(G)$, is equal to the minimum cardinality of an independent dominating set in G. Finally, S is a *total dominating set* if every vertex in V is adjacent to at least one vertex in S. The *total domination number* of a graph G, denoted by $\gamma_t(G)$, is equal to the minimum cardinality of a total dominating set in G.

One can also consider the problem of partitioning the vertex set of a given graph into any of these sets (see e.g., [14] for an overview). A *domatic k-partition* is a partition of $V(G)$ into k pairwise disjoint dominating sets. Introduced by Cockayne and Hedetniemi [5], the *domatic number* of a graph G, denoted by $\mathrm{dom}(G)$, is the maximum order k of a domatic k-partition. In addition, the *idomatic number* of a graph G, denoted by $\mathrm{idom}(G)$, equals the maximum order of a partition of $V(G)$ into independent dominating sets. Further, the *total domatic number* of a graph G, denoted by $\mathrm{tdom}(G)$, equals the maximum order of a partition of $V(G)$ into total dominating sets. The *chromatic number* of a graph G, denoted by $\chi(G)$, equals the minimum order of a partition of $V(G)$ into independent sets. The computation of any of the invariants $\mathrm{dom}(G)$, $\mathrm{idom}(G)$, $\mathrm{tdom}(G)$, and $\chi(G)$ is known to be NP-hard (see e.g., [7]).

The queens graph Q_n, for any integer $n \geq 1$, has the n^2 squares of the $n \times n$ chessboard as its vertices with two vertices adjacent if and only if they are on the same row, column or diagonal of the board. That is, the edges correspond to the legal moves by a queen. One of the most well-known problems involving the queens graph is the *n-queens problem* (see [4] and the references therein), which asks for the number of independent sets of size n in Q_n. Equivalently, in how many ways can you place n queens on the $n \times n$ chessboard so that no two queens can attack one another? The solution to this problem is only known for n up to 27 and given as sequence A000170 in the OEIS. Besides the n-queens problem, many

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other problems appear difficult on Q_n even for small values of n. For example, determining the chromatic number of Q_n poses a non-trivial computational challenge [11–13] and is known for n up to 26 as sequence A088202 in the OEIS.

Recently, Hedetniemi et al. [6] considered the domatic, idomatic, and total domatic number of Q_n. Among other results, the authors determined either exactly or gave bounds on the values of $\mathrm{dom}(Q_n)$, $\mathrm{tdom}(Q_n)$, and $\mathrm{idom}(G)$ for n up to 9. As open questions, they asked for the values of $\mathrm{dom}(Q_7)$, $\mathrm{dom}(Q_9)$, and also for the value of $\mathrm{tdom}(Q_9)$. Using a computational approach based on SAT solvers, we determine these values exactly. In particular, our results settle the value of all three invariants exactly for n up to 9. Further, we determine either exactly or give both lower and upper bounds for them on n up to 12. Our results are summarized in Table 1.1.

Table 1.1: A summary of our results with new results in bold. The values of $\gamma(Q_n)$ and $i(Q_n)$ are from A075458 and A075324, respectively, in the OEIS. The values of $\gamma_t(Q_n)$ for $n \geq 9$ are computed by us using a SAT-based approach. In particular, the value of $\gamma_t(Q_9)$ is incorrectly marked in [6, Table 5] as 6 but we correct it here to 5.

n	4	5	6	7	8	9	10	11	12
$\gamma(Q_n)$	2	3	3	4	5	5	5	5	6
$i(Q_n)$	3	3	4	4	5	5	5	5	7
$\gamma_t(Q_n)$	2	3	4	4	5	**5**	**6**	**7**	**7**
$\mathrm{dom}(Q_n)$	8	8	10	**11**	12	**13**	**16**	**18**-20	**20**-21
$\mathrm{idom}(Q_n)$	5	7	8	10	12	**14**	**15-16**	**17**	**18-20**
$\mathrm{tdom}(Q_n)$	8	8	9	11	12	**14**	**15-16**	**16-17**	**18-20**

2. Computational approach

In this section, we explain our encoding and describe the methods used.

Encodings. Let $G = (V, E)$ be a finite simple graph. Our goal is to encode the problem of deciding whether $V(G)$ can be partitioned into k dominating sets. To this end, we introduce Boolean variables $x_{v,i}$ for each $v \in V$ and $i \in \{0, 1, \ldots, k-1\}$ such that $x_{v,i}$ is true when vertex v is assigned to dominating set i. Then, we have the following clauses:

(1) (each vertex in exactly one set) $\Sigma_{i=0}^{k-1} x_{v,i} = 1$, $\forall v \in V$, and

(2) (each set is dominating) $\bigvee_{u \in N[v]} x_{u,i}$, $\forall i \in \{0, 1, \ldots, k-1\}, \forall v \in V$.

This formulation is satisfiable precisely when G can be partitioned into k dominating sets. Further, for a total domatic partition, we modify the encoding of (2) to consider the open neighborhood of vertex v. On the other hand, for an idomatic partition, we keep (1) and (2) as-is but additionally add the clause:

(3) (each set is independent) $\overline{x_{u,i}} \vee \overline{x_{v,i}}$, $\forall uv \in E$.

Here, $\overline{x_{u,i}}$ denotes the Boolean negation of $x_{u,i}$, i.e., the clause ensures that no two adjacent vertices are placed in the same set.

It should be noted that there are many ways of expressing an "exactly one" constraint such as (1) (for example, see [3, Chapter 2]). A straightforward way is combining an "at least one" clause and a set of "at most one" clauses. For instance, for a vertex v with variables $x_{v,0}$, $x_{v,1}$, …, $x_{v,k-1}$ this is given as the conjunction of

$$(x_{v,0}, x_{v,1}, \ldots, x_{v,k-1})$$

and for each pair $0 \leq i < j \leq k-1$ the clause

$$(\overline{x_{v,i}} \vee \overline{x_{v,j}}).$$

To avoid the quadratic number of clauses added by the straightforward pairwise encoding, we use the more compact sequential counter encoding [10]. For all our modeling, we use the PySAT framework [8, 9].

Similarly, many combinatorial problems have symmetries and we should strive to eliminate this redundancy by e.g., fixing some part of the solution. In our case, when finding the domatic or total domatic number, we always fix one corner square to be in the set with label zero. On the other hand, when finding the idomatic number, we pick an arbitrary maximum clique of the graph, and fix its vertices to be in different parts of the partition: this is always safe, since each set must be independent.

Constraints on the sizes of the sets. Suppose we are looking for a domatic k-partition $\mathcal{D}$ in a graph G of order n with domination number $\gamma(G)$. In order to find solutions faster or to determine no solution exists, it can be helpful to analyze the sizes of the sets in $\mathcal{D}$ and encoding this requirement into the SAT instance before running the SAT solver. To do so, we can set up the following integer feasibility program

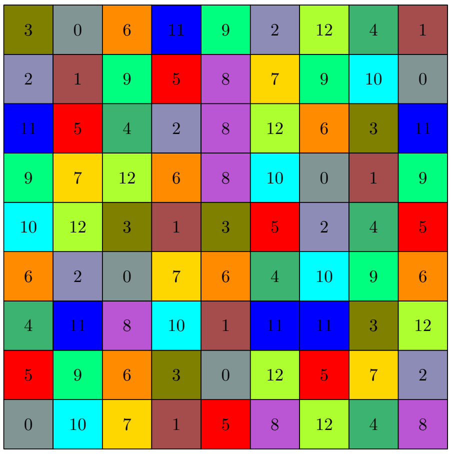
$$\begin{aligned} \text{Find} \quad & x_0, x_1, \ldots, x_{k-1} \in \mathbb{Z}^+ \\ \text{s.t.} \quad & \sum_{i=0}^{k-1} x_i = n, \\ & x_i \geq \gamma(G) \quad \text{for } i = 0, 1, \ldots, k-1. \end{aligned}$$

That is, the value for x_i means the set $D_i \in \mathcal{D}$ has size x_i. Especially when proving that no domatic k-partition exists, explicitly enumerating all the unlabeled solutions to this problem is crucial to obtaining a solution in reasonable time from the SAT solver. Later on in our proofs, whenever we claim that a specific number of cases must be considered to rule out the existence of a partition into k sets, the argument is based on enumerating the unlabeled solutions to this program, where $\gamma(G)$ can be replaced by $i(G)$ or $\gamma_t(G)$, as required.

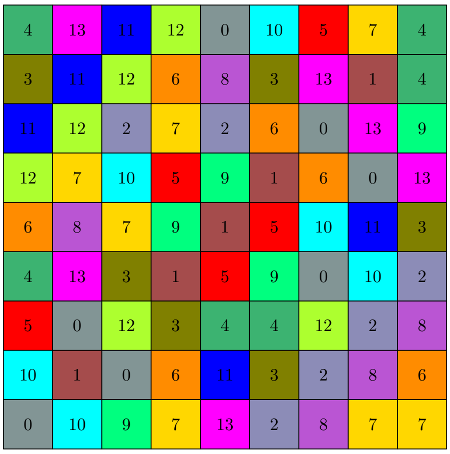
Solvers. The first SAT solver we use is CaDiCaL [2], which is a complete solver. That is, given enough time and memory, the solver is able to prove both the satisfiability and unsatisfiability of an instance. The other solver we use is Sparrow [1], which is a stochastic local search solver. Sparrow is incomplete, meaning it cannot prove the unsatisfiability of a formula. For satisfiable instances, it can sometimes find solutions in an instant compared to the longer runtime of CaDiCaL.

3. Exact values and improved bounds

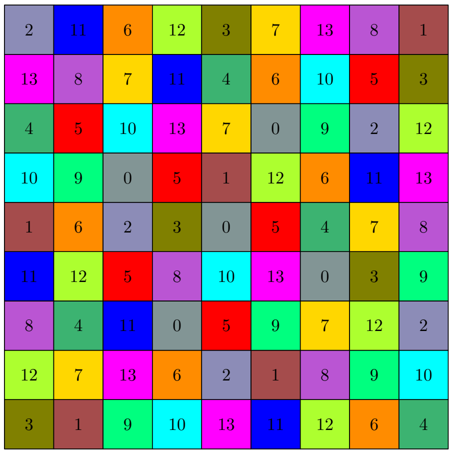
All our results are obtained by using the described computational approach using SAT solvers. The SAT solver is executed on a machine equipped with an AMD Ryzen 9 3900X CPU and 32 GB of memory. For all our SAT solver runs, we use a timeout of 8 hours but make no attempt at optimizing the runtime further.



(a) $\mathrm{dom}(Q_9) = 13$



(b) $\mathrm{tdom}(Q_9) = 14$



(c) $\mathrm{idom}(Q_9) = 14$

Figure 3.1: Tighter results for the queens graph Q_9.

Small values of n. We begin by considering small values of $n \leq 9$ for which exact values or bounds were tabulated in [6, Table 5]. Among their open problems, the authors explicitly asked for the values of $\mathrm{dom}(Q_7)$, $\mathrm{dom}(Q_9)$, and $\mathrm{tdom}(Q_9)$ which we settle precisely here.

Theorem 3.1. *The queens graph Q_7 has $\mathrm{dom}(Q_7) = 11$.*

Proof. Hedetniemi et al. [6, Figure 6] gave an explicit construction to show that $\mathrm{dom}(Q_7) \geq 11$. To prove that $\mathrm{dom}(Q_7) = 11$, we must show that a domatic 12-partition of Q_7 does not exist. First, we know that $\gamma(Q_7) = 4$, meaning that each set of a domatic 12-partition must have size at least 4. On the other hand, the sum of the cardinalities of the sets in the partition must sum to 49. Therefore, the only possibility is to have exactly one set of size 5, and the remaining eleven sets of size 4. We add information about this forced structure into our SAT formula by adding the clauses $\sum_{v \in V} x_{v,0} = 5$ and $\sum_{v \in V} x_{v,i} = 4$ for each $i \in \{1, 2, \ldots, 11\}$, encoded using standard methods. With this enhanced encoding the solver reports UNSAT, meaning there is no solution. The result follows. □

Theorem 3.2. *The queens graph* Q_9 *has* $\mathrm{dom}(Q_9) = 13$.

Proof. The analysis of the structure of any domatic partition of Q_9 by Hedetniemi et al. [6] shows that $\mathrm{dom}(Q_9) \leq 13$. We give a matching lower bound by demonstrating a domatic 13-partition of Q_9 in Figure 3.1a. Thus, the result follows. □

The value of $\mathrm{tdom}(Q_n)$ was settled in [6] for n up to 8. For $n = 9$, the authors claimed that $\mathrm{tdom}(Q_9) \geq 11$. We improve upon this bound considerably by showing that a total domatic 14-partition exists, and this is in fact optimal.

Theorem 3.3. *The queens graph* Q_9 *has* $\mathrm{tdom}(Q_9) = 14$.

Proof. The construction given in Figure 3.1b shows that $\mathrm{tdom}(Q_9) \geq 14$. From [6, Table 5], we know that $\mathrm{tdom}(Q_9) \leq 15$. Now, let us prove that a total domatic 15-partition $\mathcal{D}$ does not exist. As $\gamma_t(Q_9) = 5$ (see Table 1.1), there are eleven cases to consider, written in vector form:

1. (5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 11),
2. (5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 6, 10),
3. (5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 7, 9),
4. (5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 8, 8),
5. (5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 6, 6, 9),
6. (5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 6, 7, 8),
7. (5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 7, 7, 7),
8. (5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 6, 6, 6, 8),
9. (5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 6, 6, 7, 7),
10. (5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 7), and
11. (5, 5, 5, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6).

For each case, we encode the corresponding size requirement for the sets D_i. For each case, the solver reports UNSAT. We conclude that no such $\mathcal{D}$ exists, so $\mathrm{tdom}(Q_9) = 14$. □

In [6], it was claimed without explicit proof that an idomatic 11-partition exists for Q_9. We improve upon this claim by constructing an idomatic 14-partition, and by proving that it is optimal.

Theorem 3.4. *The queens graph* Q_9 *has* $\mathrm{idom}(Q_9) = 14$.

Proof. The construction shown in Figure 3.1c gives us that $\mathrm{idom}(Q_9) \geq 14$. Let us then prove that an idomatic 15-partition does not exist for the Q_9. Similarly to the proof of Theorem 3.3, we must consider exactly the same set sizes, i.e., eleven cases. For each case, the SAT solver reports UNSAT. It follows that $\mathrm{idom}(Q_9) = 14$. □

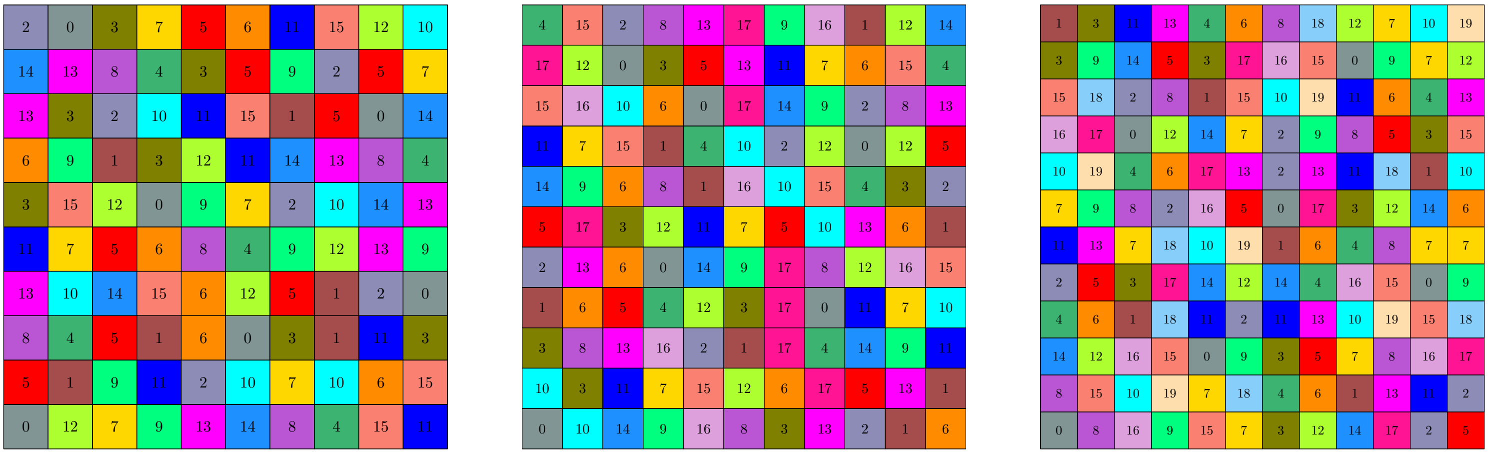
Large values of *n*. We then turn to larger values of $n > 9$. The values of $\mathrm{dom}(Q_n)$ for $n \in \{10, 11, 12\}$ were discussed in [6], but only upper bounds of the form $\lfloor n^2/\gamma(Q_n) \rfloor$ were given. Before proceeding, the following lemma will be useful to simplify some of our proofs.

Lemma 3.1 (Hedetniemi et al. [6]). *There can be only four minimum dominating sets in any domatic partition of* Q_{10} *or* Q_{11}.

Theorem 3.5. *The queens graph* Q_{10} *has* $\mathrm{dom}(Q_{10}) = 16$.

Proof. The construction shown in Figure 3.2a establishes that $\mathrm{dom}(Q_{10}) \geq 16$. To prove that no domatic 17-partition exists, we proceed similarly to the proof of Theorem 3.3. This time, from Table 1.1, we have that $\gamma_t(Q_{10}) = 5$ and there are 176 cases to consider. By Lemma 3.1, it suffices to verify only four cases out of 176:

1. (5, 5, 5, 5, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 8)
2. (5, 5, 5, 5, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 7, 7)
3. (5, 5, 5, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 7), and
4. (5, 5, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6).



(a) $\text{dom}(Q_{10}) = 16$ **(b)** $\text{dom}(Q_{11}) \geq 18$ **(c)** $\text{dom}(Q_{12}) \geq 20$

Figure 3.2: Improved results on $\text{dom}(Q_n)$ for $n \in \{10, 11, 12\}$.

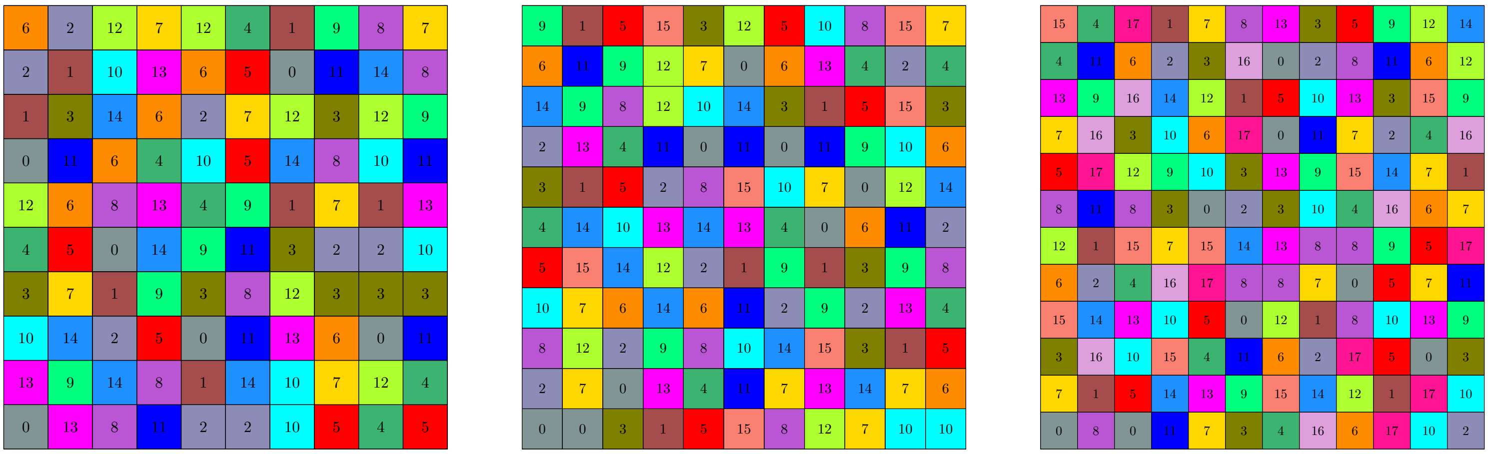
The SAT solver reports UNSAT for each case, so we conclude that $\text{dom}(Q_{10}) = 16$. □

Theorem 3.6. *The queens graph Q_n has*

$$\text{dom}(Q_n) \in \begin{cases} \{18, 19, 20\}, & \textit{if } n = 11, \\ \{20, 21\}, & \textit{if } n = 12. \end{cases}$$

Proof. The upper bounds for both n follow by computing $\lfloor n^2/\gamma(Q_n) \rfloor$. To conclude the proof, the lower bounds follow from the constructions given in Figure 3.2b and Figure 3.2c, respectively. □

Next, we give tighter bounds for $\text{tdom}(Q_n)$ for $n \leq 12$. To do that, we first compute $\gamma_t(Q_n)$ using a SAT solver to obtain upper bounds via $\lfloor n^2/\gamma_t(Q_n) \rfloor$. Again, these upper bounds are complemented by almost matching lower bounds.



(a) $\text{tdom}(Q_{10}) \geq 15$ **(b)** $\text{tdom}(Q_{11}) \geq 16$ **(c)** $\text{tdom}(Q_{12}) \geq 18$

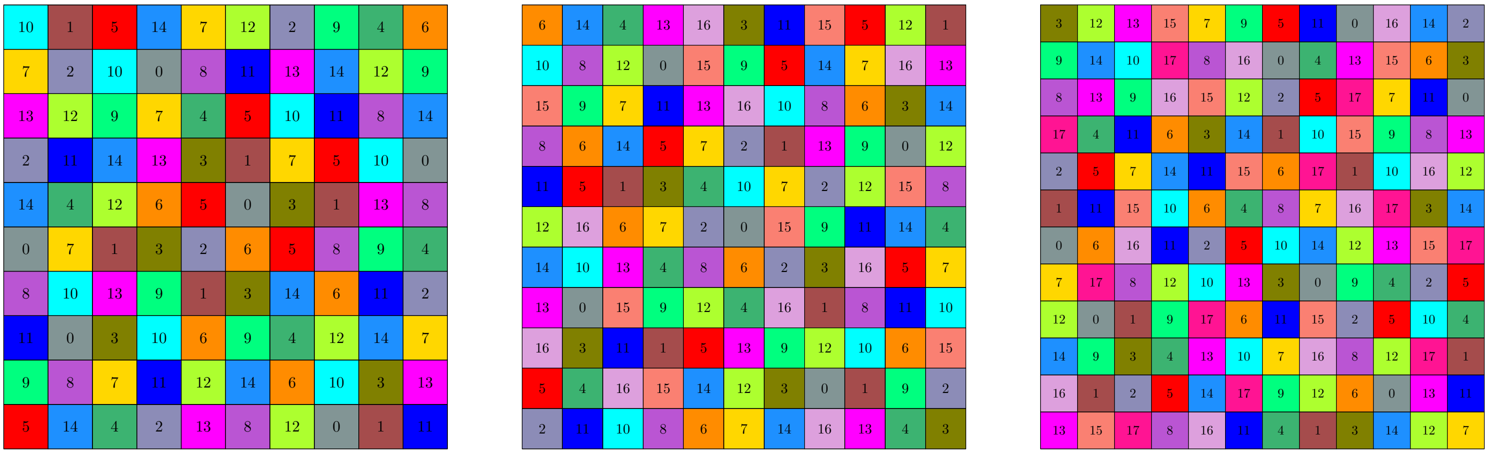
Figure 3.3: Improved bounds on $\text{tdom}(Q_n)$ for $n \in \{10, 11, 12\}$.

Theorem 3.7. *The queens graph Q_n has*

$$\text{tdom}(Q_n) \in \begin{cases} \{15, 16\}, & \textit{if } n = 10, \\ \{16, 17\}, & \textit{if } n = 11, \\ \{18, 19, 20\}, & \textit{if } n = 12. \end{cases}$$

Proof. For $n \in \{10, 11, 12\}$, the computed upper bounds are almost matched by the lower bounds from our constructions in Figure 3.3a, Figure 3.3b, and Figure 3.3c, respectively. □

Let us now consider the idomatic number. Clearly, the chromatic number $\chi(G)$ of a graph G is a lower bound for its idomatic number $\text{idom}(G)$. In this regard, Vasquez [11] determined that $\chi(Q_{10}) = 11$, $\chi(Q_{11}) = 11$, and $\chi(Q_{12}) = 12$. We can improve upon these lower bounds greatly.



(a) $\mathrm{idom}(Q_{10}) \geq 15$ **(b)** $\mathrm{idom}(Q_{11}) = 17$ **(c)** $\mathrm{idom}(Q_{12}) \geq 18$

Figure 3.4: Improved results on $\mathrm{idom}(Q_n)$ for $n \in \{10, 11, 12\}$.

Theorem 3.8. *The queens graph* Q_{10} *has* $15 \leq \mathrm{idom}(Q_{10}) \leq 16$.

Proof. The construction given in Figure 3.4a gives us that $\mathrm{idom}(Q_{10}) \geq 15$. As in the proof of Theorem 3.5, we have 176 cases to check to prove that an idomatic 17-partition does not exist. For each case, the SAT solver reports UNSAT, and the claim follows. □

Theorem 3.9. *The queens graph* Q_{11} *has* $\mathrm{idom}(Q_{11}) = 17$.

Proof. Our construction given in Figure 3.4b shows that $\mathrm{idom}(Q_{11}) \geq 17$. To prove that this is optimal, we must show that an idomatic 18-partition does not exist. This time, there are 6570 cases to check. For each case, the SAT solver reports UNSAT, so the result follows. □

Theorem 3.10. *The queens graph* Q_{12} *has* $18 \leq \mathrm{idom}(Q_{12}) \leq 20$.

Proof. Our construction given in Figure 3.4c shows that $\mathrm{idom}(Q_{12}) \geq 18$. On the other hand, the upper bound follows from $\mathrm{idom}(Q_{12}) \leq \lfloor 144/7 \rfloor = 20$ concluding the proof. □

4. Directions for further research

Besides the remaining open problems mentioned by Hedetniemi et al. [6], one may also consider determining the domatic numbers of $m \times n$ chessboards. Alternatively, other highly structured graphs, such as cylinder or torus graphs, could be studied. Finally, general constructions for any of the three domatic numbers that yield good bounds for the queens graph are also of interest.

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