Discrete Mathematics Letters
www.dmlett.com

Discrete Math. Lett. **11** (2023) 31–37
DOI: 10.47443/dml.2022.119

Research Article

Irregular domination trees and forests

Caryn Mays, Ping Zhang*

Department of Mathematics, Western Michigan University, Kalamazoo, Michigan 49008-5248, USA

(Received: 19 July 2022. Accepted: 21 September 2022. Published online: 28 September 2022.)

Abstract

A set S of vertices in a connected graph G is an irregular dominating set if the vertices of S can be labeled with distinct positive integers in such a way that for every vertex v of G, there is a vertex $u \in S$ such that the distance from u to v is the label assigned to u. If for every vertex $u \in S$, there is a vertex v of G such that u is the only vertex of S whose distance to v is the label of u, then S is a minimal irregular dominating set. A graph H is an irregular domination graph if there exists a graph G with a minimal irregular dominating set S such that H is isomorphic to the subgraph $G[S]$ of G induced by S. In this paper, all irregular domination trees and forests are characterized. All disconnected irregular domination graphs are determined as well.

Keywords: distance; irregular domination; irregular domination graph; trees; forests.

2020 Mathematics Subject Classification: 05C05, 05C12, 05C69, 05C76.

1. Introduction

A set S of vertices in a nontrivial connected graph G is called an *irregular dominating set* if the vertices of S can be labeled with distinct positive integers in such a way that for every vertex v in G, there is at least one vertex $u \in S$ such that the distance $d(u, v)$ between u and v is the label $\ell(u)$ assigned to u. Thus, no label can be greater than the diameter $\mathrm{diam}(G)$ of G (the greatest distance between any two vertices of G). The vertex u is said to *dominate* all vertices v for which $d(u, v) = \ell(u)$. Such a labeling is called an *irregular dominating labeling*. This concept was introduced and studied in [4] and studied further in [2, 3, 5]. More generally, irregularity in graphs is discussed in [1] and major results on graph domination are presented in [6] by Haynes, Hedetniemi, and Henning.

When considering an irregular dominating set S in a connected graph G, it is assumed that the vertices of S have been assigned distinct positive integer labels. In [4], all trees having an irregular dominating set are determined. A path of order n is denoted by P_n and a star is a tree of diameter 2.

Theorem 1.1. *A nontrivial tree T has an irregular dominating set if and only if T is none of P_2, P_6 or a star.*

If G is a connected graph possessing an irregular dominating set, then the minimum cardinality of an irregular dominating set in G is the *irregular domination number* $\tilde{\gamma}(G)$ of G. If G is such a connected graph of diameter d, then an irregular dominating labeling of G uses labels from the set $[d] = \{1, 2, \ldots, d\}$ and so $\tilde{\gamma}(G) \leq d$.

An irregular dominating set S in a graph is *minimal* if for every vertex $u \in S$, there is a vertex v of G such that u is the only vertex that dominates v. In [7], the structural relationships of minimal irregular dominating sets in certain well-known graphs are studied, which led to the concept of an irregular domination graph. A graph H is an *irregular domination graph* if there exists a graph G possessing a minimal irregular dominating set S such that the subgraph $G[S]$ of G induced by S is isomorphic to H. As we saw in Theorem 1.1, the path P_6 does not have an irregular dominating set. Nevertheless, it is an irregular domination graph, as shown by the graph G of Figure 1. For the set $S = \{u_1, u_2, \ldots, u_6\}$, the vertex u_1 is the only vertex of S that dominates u_4, the vertex u_2 is the only vertex of S that dominates u_3, the vertex u_3 is the only vertex of S that dominates x, the vertex u_4 is the only vertex of S that dominates u_2, the vertex u_5 is the only vertex of S that dominates y, and the vertex u_6 is the only vertex of S that dominates z, Therefore, S is a minimal irregular dominating set and $G[S] \cong P_6$.

In [7], the next three propositions were obtained and were used to establish Theorem 1.2 given on the next page.

Proposition 1.1. *No connected graph of diameter at most 2 is an irregular domination graph.*

*Corresponding author (ping.zhang@wmich.edu).

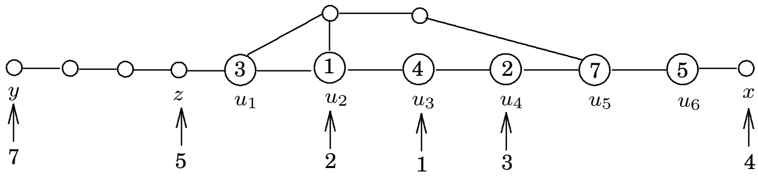

Figure 1: Showing that P_6 is an irregular domination graph.

By Proposition 1.1, no star is an irregular domination graph.

Proposition 1.2. *The graph $K_1 + K_2$ is the only irregular domination graph of order 3.*

Proposition 1.3. *A graph H of order 4 or 5 is an irregular domination graph if and only if H is disconnected, or H is connected and* $\operatorname{diam}(H) \geq 3$.

Theorem 1.2. *A path P_n of order $n \geq 2$ is an irregular domination graph if and only if $n \geq 4$.*

It was shown in [7] that neither the 3-cube $Q_3 = C_4 \,\square\, K_2$ nor the prism $C_5 \,\square\, K_2$ are irregular domination graphs. Both of these graphs have diameter 3. It was stated as an open question in [7] whether every connected graph of diameter 4 or more is an irregular domination graph. By Theorem 1.2, every path of diameter at least 3 is an irregular domination graph. It is our goal here to determine all irregular domination trees and forests. In addition, we determine all disconnected irregular domination graphs.

2. Irregular domination forests

In [7], it was shown that if an isolated vertex is added to any graph of order 3 or more, then the resulting disconnected graph is an irregular domination graph.

Theorem 2.1. *If H is a graph of order 3 or more, then $H + K_1$ is an irregular domination graph.*

The following is a consequence of Theorem 2.1.

Corollary 2.1. *If F is a forest of order 4 or more having an isolated vertex, then F is an irregular domination graph.*

The following result gives another sufficient condition for a forest to be an irregular domination graph.

Theorem 2.2. *If F is a forest containing a component of diameter 3 or more, then F is an irregular domination graph.*

Proof. Let F be a forest of order $n \geq 4$ containing a component of diameter 3 or more. Then F contains a path $P = (u_1, u_2, u_3, u_4)$, where u_1 is an end-vertex of F. We consider two cases depending on whether $\deg_F u_2 = 2$ or $\deg_F u_2 \geq 3$.

Case 1. $\deg_F u_2 = 2$. Let $X = V(F) - V(P)$. Let G be the graph obtained from F by adding (a) a path $(y_1, y_2, \ldots, y_{n-1})$ of order $n-1$ and joining y_1 to u_2 and (b) a vertex z and joining z to every vertex in $\{u_2\} \cup X$. Then $d_G(x, u_2) = 2$ and the diameter of G is $\operatorname{diam}(G) = d(x, y_{n-1}) = 2+(n-1) = n+1$ for each $x \in X$. Let $S = V(F)$. We define a labeling $f : S \to [n+1]$ of G as follows:

- ⋆ $f(u_1) = 3$, $f(u_2) = 1$, $f(u_3) = 4$, $f(u_4) = 2$, and
- ⋆ the $n-4$ labels in the set $[6, n+1]$ are assigned arbitrarily to the $n-4$ vertices in X.

Hence, the set of labels assigned to the vertices of S by f is $[n+1] - \{5\}$. It remains to show that S is a minimal irregular dominating set S. Since

(1) each vertex in $V(F)$ is dominated by a vertex labeled i for some $i \in [3]$ and u_2 is only dominated by the vertex labeled 2,

(2) the vertex z is only dominated by the vertex u_2 labeled 1,

(3) the vertex y_1 is only dominated by the vertex labeled 1, the vertex y_2 is only dominated by the vertex labeled 3, the vertex y_3 is only dominated by the vertex labeled 4, and

(4) for $4 \leq i \leq n-1$, the vertex y_i is only dominated by the vertex labeled $i+2$,

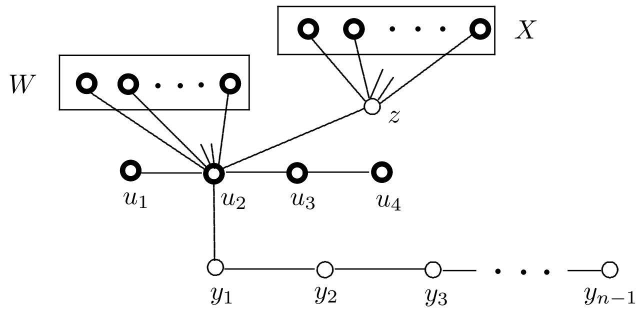

Figure 2: The graph G in the proof of Case 2 of Theorem 2.2.

it follows that S is a minimal irregular dominating set of G and $G[S] \cong F$.

Case 2. $\deg_F u_2 = t \geq 3$. Let $W = N_F(u_2) - \{u_1, u_3\}$ be the set of those $t-2$ neighbors of u_2 that do not belong to P and let $X = V(F) - (N_T(u_2) \cup \{u_2, u_4\})$. Then $|X| = n - (t+2) = n - t - 2$. The graph G is obtained from F by adding (a) a path $(y_1, y_2, \ldots, y_{n-1})$ of order $n-1$ and joining y_1 to u_2 and (b) a vertex z and joining z to every vertex in $\{u_2\} \cup X$. The graph G is shown in Figure 2. Thus, $d_G(x, u_2) = 2$ and $\operatorname{diam}(G) = d(x, y_{n-1}) = 2 + (n-1) = n+1$ each $x \in X$.
Let $S = V(F)$. We define a labeling $f : S \to [n+1]$ of G as follows:

- ⋆ $f(u_1) = 3$, $f(u_2) = 1$, $f(u_3) = 4$, $f(u_4) = 2$,
- ⋆ the $t-2$ labels in the set $[5, t+2]$ are assigned arbitrarily to the $t-2$ vertices in $W = N_T(u_2) - \{u_1, u_3\}$, and
- ⋆ the $n-t-2$ labels in the set $[t+4, n+1]$ are assigned arbitrarily to the $n-t-2$ vertices in X.

Hence, the set of labels assigned to the vertices of S by f is $[n+1] - \{t+3\}$. It remains to show that S is a minimal irregular dominating set S. Since

(1) each vertex in $V(F)$ is dominated by a vertex labeled i for some $i \in [3]$ and u_2 is only dominated by the vertex labeled 2,

(2) the vertex z is only dominated by the vertex u_2 labeled 1,

(3) the vertex y_1 is only dominated by the vertex labeled 1, the vertex y_2 is only dominated by the vertex labeled 3, the vertex y_3 is only dominated by the vertex labeled 4,

(4) for $4 \leq i \leq t+1$, the vertex y_i is only dominated by the vertex labeled $i+1$, and

(5) for $t+2 \leq i \leq n-1$, the vertex y_i is only dominated by the vertex labeled $i+2$,

it follows that S is a minimal irregular dominating set of G and $G[S] \cong F$. □

Since no tree of diameter 1 or 2 is an irregular domination graph, the following result is a consequence of Theorem 2.2.

Corollary 2.2. *A tree T is an irregular domination graph if and only if* $\operatorname{diam}(T) \geq 3$.

By Theorem 2.2, only those forests having at least two components, each of which is either K_2 or a star, remain to be considered. If every component of a disconnected forest F is K_2, then F is an irregular domination graph, as we show next.

Theorem 2.3. *If $F = kK_2$ for some integer $k \geq 2$, then F is an irregular domination graph.*

Proof. For an integer $k \geq 2$, let $E(kK_2) = \{u_iv_i : 1 \leq i \leq k\}$ be the set of the k edges of $2K_2$. We show that there exists a graph G_k with a minimal dominating set S_k such that $G_k[S_k] \cong kK_2$ (although $2K_2$ is an irregular domination graph by Proposition 1.3). For $k = 2$, let G_2 be the graph of diameter 5 shown in Figure 3. Let $S_2 = \{u_1, u_2, v_1, v_2\}$ where the corresponding labeling $f_2 : S_2 \to \{1, 2, 3, 4\}$ is also shown in Figure 3. Since (1) each vertex in $V(2K_2)$ is dominated by a vertex labeled i for some $i \in [3]$, (2) the vertex z is only dominated by the vertex labeled 1, the vertex x is only dominated by the vertex labeled 3 and the vertex y_1 is only dominated by the vertex labeled 2, and the vertex y_2 is only dominated by the vertex labeled 4, it follows that S_2 is a minimal dominating set of G_2 and $G_2[S_2] \cong 2K_2$. Thus, $2K_2$ is an irregular domination graph.

For $k \geq 3$, let H_i be a triangle with vertex set $\{u_i, v_i, w_i\}$ for $3 \leq i \leq k$. We construct a graph G_k from the graph G_2 and the triangles H_i ($3 \leq i \leq k$) by (a) joining the vertex w_i of H_i to the vertex x of G_2 for $3 \leq i \leq k$ and (b) adding a path $(y_3, y_4, \cdots, y_{2k-2})$ of order $2k-4$ and joining y_3 to y_2. Thus, $\operatorname{diam}(G_k) = d_G(v_1, y_{2k-2}) = 2k+1$. Let $S_k = V(kK_2)$ and $W = \{u_i, v_i : 3 \leq i \leq k\}$.

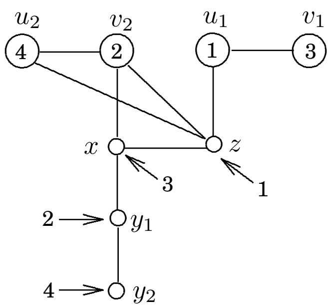

Figure 3: The graph G_2 in the proof of Theorem 2.3.

We define a labeling $f_k : S_k \to [2k]$ as follows:

- ⋆ $f_k(v) = f_2(v)$ if $v \in V(G_2)$ and
- ⋆ the $2k-4$ labels in the set $[5, 2k]$ are assigned arbitrarily to the $2k-4$ vertices in W.

Hence, the set of labels assigned to the vertices of S_k by f_k is $[2k]$. It remains to show that S_k is a minimal dominating set of G_k. Since

(1) each vertex in $V(kK_2)$ is dominated by a vertex labeled i for some $i \in [4]$,

(2) the vertex z is only dominated by the vertex labeled 1, the vertex x is only dominated by the vertex labeled 3 and the vertex y_1 is only dominated by the vertex labeled 2, and

(3) for $2 \le i \le 2k-2$, the vertex y_i is only dominated by the vertex labeled $i+2$,

it follows that S_k is a minimal irregular dominating set of G_k and $G_k[S_k] \cong 2K_2$. □

By Theorems 2.2 and 2.3, only one situation remains, namely when all components of a disconnected forest are stars or K_2 and at least one component is a star.

Theorem 2.4. *Let $F = T_1 + T_2 + \cdots + T_k$ be a forest with $k \ge 2$ components T_i where $1 \le i \le k$, each of which is either K_2 or a star. If F contains at least one star, then F is an irregular domination graph.*

Proof. Let $F = T_1 + T_2 + \cdots + T_k$ be a forest of order n, where T_1 is a star and each tree T_i, $2 \le i \le k$, is either K_2 or a star and $u_i \in V(T_i)$ such that $\deg_{T_i} u_i = \Delta(T_i)$ for $1 \le i \le k$. Thus, if T_i is a star, then u_i is the center of T_i for $1 \le i \le k$. We consider two cases, depending on $k = 2$ or $k \ge 3$.

Case 1. $k = 2$. Let G be obtained from F by adding a path $(y_1, y_2, \ldots, y_{n-1})$ of order $n-1$ and joining y_1 to u_1 and u_2. Let $v_{1,1}$ and $v_{1,2}$ be two neighbors of u_1 in T_1 and let $v_{2,1}$ be a neighbor of u_2 in T_2. Then the diameter of G is $\operatorname{diam}(G) = d_G(v_{1,1}, y_{n-1}) = n$. Let $X = V(F) - \{u_1, u_2, v_{1,1}, v_{1,2}, v_{2,1}\}$. Then $|X| = n-5$. Let $S = V(F)$. We define a labeling $f : S \to [n]$ as follows:

- ⋆ $f(u_1) = 1$, $f(u_2) = 2$, $f(v_{1,1}) = 3$, $f(v_{1,2}) = 4$, $f(v_{2,1}) = 5$ and
- ⋆ the $n-5$ labels in $[6, n]$ are assigned arbitrarily to the $n-5$ vertices of X.

Hence, the set of labels assigned to the vertices of S by f is $[n]$. It remains to show that S is a minimal irregular dominating set of G. Since

(1) each vertex in $V(F)$ is dominated by a vertex labeled i for some $i \in [4]$, the vertex u_1 is only dominated by the vertex labeled 2, and the vertex u_2 is only dominated by the vertex labeled 3,

(2) the vertex y_1 is only dominated by the vertex labeled 1 and the vertex y_2 is dominated by the vertex labeled 2 or 3, and

(3) for $3 \le i \le n-1$, the vertex y_i is only dominated by the vertex labeled $i+1$.

it follows that S is a minimal irregular dominating set of G and $G[S] \cong F$.

Case 2. $k \ge 3$. Let G be obtained from F by adding (a) a vertex z and joining z to every vertex in $\{u_3, u_4, \ldots, u_k\}$ and (b) a path $(y_1, y_2, \ldots, y_n)$ of order n and joining y_1 to each vertex in $\{z, u_1, u_2\}$. Let $v_{1,1}$ and $v_{1,2}$ be two neighbors of u_1 in T_1 and let

$v_{2,1}$ be a neighbor of u_2 in T_2. Then the diameter of G is $\text{diam}(G) = d_G(v_{3,1}, y_n) = n+1$, where $v_{3,1}$ is a neighbor of u_3 in T_3. Let $W = V(F) - (V(T_1) \cup V(T_2) \cup \{u_3, u_4, \ldots, u_k\})$ and $X = V(T_1) \cup V(T_2) - \{u_1, u_2, v_{1,1}, v_{1,2}, v_{2,1}\}$. Let $p = |V(T_1) \cup V(T_2)|$. Then $|W| = p - 5$ and $|X| = n - p - (k-2) = n - p - k + 2$. Let $S = V(F)$. We define a labeling $f : S \to [n+1]$ as follows:

- ⋆ $f(u_1) = 1$, $f(u_2) = 2$, $f(v_{1,1}) = 3$, $f(v_{1,2}) = 4$, $f(v_{2,1}) = 5$,
- ⋆ the $p-5$ labels in $[6, p]$ are assigned arbitrarily to the $p-5$ vertices of X,
- ⋆ the $k-2$ labels in $[p+1, p+(k-2)]$ are assigned arbitrarily to the $k-2$ vertices of $\{u_3, u_4, \ldots, u_k\}$, and
- ⋆ the $n-p-k+2$ labels in $[p+k, n+1]$ are assigned arbitrarily to the $n-p-k+2$ vertices of W.

Hence, the set of labels assigned to the vertices of S by f is $[n+1] - \{p+k-1\}$. This is shown in Figure 4 where $T_1 = K_{1,3}$, $T_2 = K_2$, $T_3 = K_{1,2}$, and $T_4 = K_{1,3}$. Thus, $k = 4$, $p = 6$, and $n = 13$.

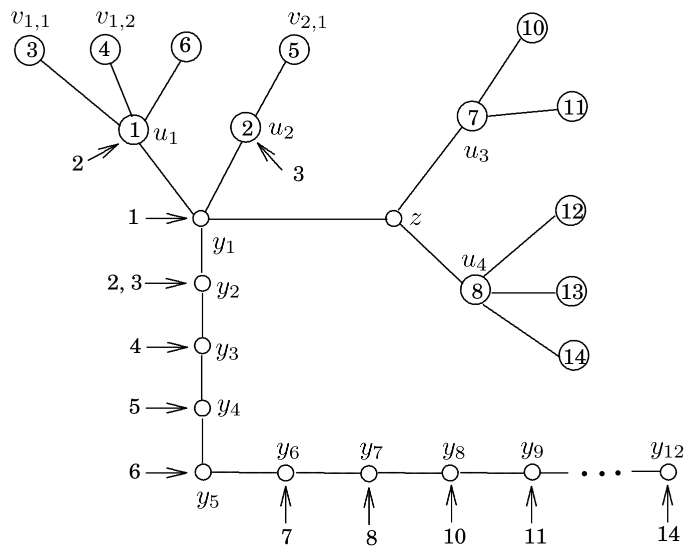

Figure 4: The graph G in the proof of Case 2 of Theorem 2.4.

It remains to show that S is a minimal irregular dominating set of G. Since

(1) each vertex in $V(F)$ is dominated by a vertex labeled i for some $i \in [4]$ and u_2 is only dominated by the vertex labeled 2,

(2) the vertex z is only dominated by the vertex u_2 labeled 1,

(3) the vertex y_1 is only dominated by the vertex labeled 1, the vertex y_2 is only dominated by the vertex labeled 3, the vertex y_3 is only dominated by the vertex labeled 4,

(4) for $4 \le i \le t+1$, the vertex y_i is only dominated by the vertex labeled $i+1$, and

(5) for $t+2 \le i \le n-1$, the vertex y_i is only dominated by the vertex labeled $i+2$,

it follows that S is a minimal irregular dominating set of G and $G[S] \cong F$. □

Observe that the graph constructed in each case of the proof of Theorems 2.4 is a tree. Thus, if F is a disconnected forest in which each component is either K_2 or a star and at least one component is a star, then there is a tree T with a minimal irregular dominating set S such that $T[S] \cong F$. We are now able to characterize all forests that are irregular domination graphs.

Corollary 2.3. *A forest F is an irregular domination graph if and only if*

(1) *F is a tree of diameter 3 or more,*

(2) *$F \cong K_1 + K_2$ or F is disconnected of order 4 or more.*

3. Disconnected irregular domination graphs

As a consequence of Theorem 2.1, Corollary 2.2, and arguments used in the proofs of Theorems 2.2, 2.3, and 2.4, every disconnected graph of order 4 or more, in which at least one component is a tree, is an irregular domination graph. In fact, more can be said about disconnected irregular domination graphs in general.

Theorem 3.1. *A disconnected graph in which at least one component has order* 3 *or more is an irregular domination graph.*

Proof. Let $H = H_1 + H_2 + \cdots + H_k$ be a disconnected graph of order n consisting of $k \geq 2$ components $H_1, H_2, \ldots, H_k$, where H_1 has order 3 or more. Let $u \in V(H_1)$ such that $\deg_{H_1} u \geq 2$ and let $w \in V(H_2)$. Next, let $W = V(H) - V(H_1)$ and let $X = V(H_1) - N[u]$, where possibly $X = \emptyset$. A graph G is constructed from H by adding

(a) a vertex z and joining z to each vertex in W and

(b) a path $(z_1, z_2, y_1, y_2, \ldots, y_{n-2})$ of order n and joining z_1 to each vertex in $\{u, w\} \cup X$ and joining z_2 to u and z.

The graph G is shown in Figure 5, where any edge joining a vertex of $N(u)$ and a vertex of X is not drawn as well as any edge joining vertices in X, $N(u)$, or W. The diameter of G is $\text{diam}(G) = d_G(w, y_{n-2}) = n$ for each $w \in W$.

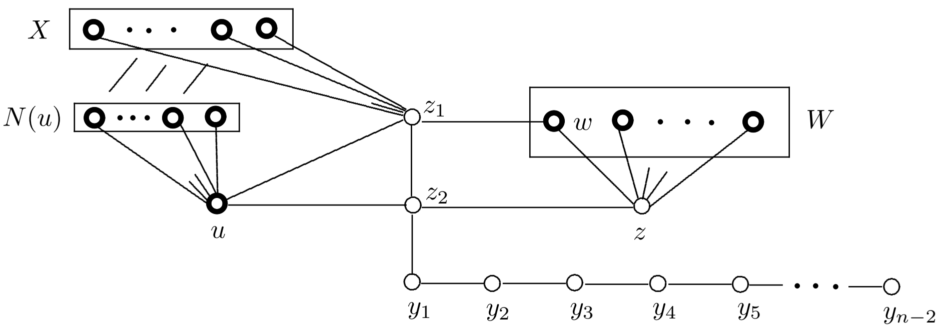

Figure 5: The graph G in the proof of Theorem 3.1.

Let u_1 and u_2 be two neighbors of u. We define a labeling $f : V(H) \to [n]$ of G by

$$f(u) = 1,\ f(w) = 2,\ f(u_1) = 3,\ f(u_2) = 4.$$

The $n-4$ labels in the set $[5, n]$ are assigned arbitrarily to the $n-4$ vertices in the set $V(H) - \{u, u_1, u_2, w\}$. Thus, the set of labels assigned to the vertices of $V(H)$ by f is $[n]$. The graph G is shown in Figure 6 for a graph H of order $n = 10$, where any edge joining a vertex of $N(u)$ and a vertex of X is not drawn as well as any edge joining vertices in X, $N(u)$, or W.

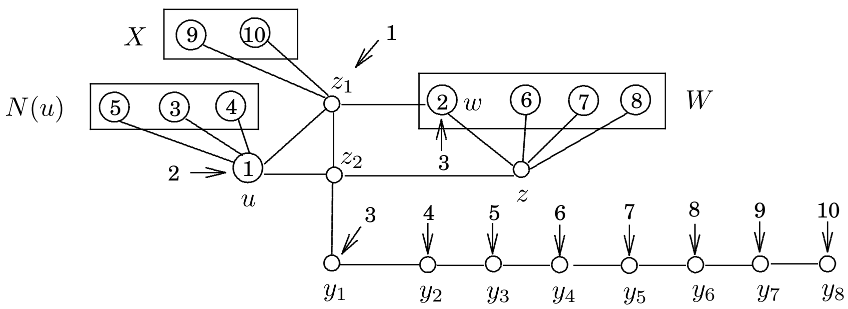

Figure 6: The graph G in the proof of Theorem 3.1.

It remains to show that S is a minimal irregular dominating set of G. Since

(1) each vertex in $V(H)$ is dominated by a vertex labeled i for some $i \in [4]$, each vertex in X is dominated by the vertex labeled 2, the vertex u is only dominated by the vertex labeled 2, and the vertex w is only dominated by the vertex labeled 3,

(2) the vertex z_1 is only dominated by the vertex labeled 1 and the vertex z_2 is dominated by the vertex labeled 1, and

(3) for $1 \leq i \leq n-2$, the vertex y_i is only dominated by the vertex labeled $i+2$,

it follows that S is a minimal irregular dominating set of G and $G[S] \cong H$. □

Observe that Theorem 2.4 is, in fact, a corollary of Theorem 3.1. The following is a consequence of Theorems 2.1, 2.3, and 3.1.

Corollary 3.1. *Every disconnected graph of order 4 or more is an irregular domination graph.*

Proof. Let G be a disconnected graph of order 4 or more. If G contains an isolated vertex, then G is an irregular domination graph by Theorem 2.1. Thus, we may assume that every component of G has order 2 or more. If every component of G is K_2, then G is an irregular domination graph by Theorem 2.3. If at least component of G has order 3 or more, then G is an irregular domination graph by Theorem 3.1. □

Since (a) there is no irregular domination graph of order 2, (b) the graph $K_2 + K_1$ is the only irregular domination graph of order 3 by Proposition 1.2, and (c) every disconnected graph of order 4 or more is an irregular domination graph by Corollary 3.1, we are now able to characterize all disconnected graphs that are irregular domination graphs.

Theorem 3.2. *A disconnected graph G is an irregular domination graph if and only if G is neither $2K_1$ nor $3K_1$.*

4. Closing comments

By Proposition 1.1, if G is a connected graph with $\mathrm{diam}(G) \le 2$, then G is not an irregular domination graph. The following result obtained in [7] gives an infinite class of graphs of diameter 3 that are not irregular domination graphs.

Theorem 4.1. *Let H be an r-regular graph, $r \ge 2$, of diameter 3 with the property that for each vertex x of H, there is exactly one vertex y such that $d(x, y) = 3$. Then H is not an irregular domination graph.*

By Corollary 2.2, every tree of diameter 3 (a double star) is an irregular domination graph and so there is an infinite class of connected graphs of diameter 3 that are irregular domination graphs. In fact, more can be said. The *eccentricity* $e(v)$ of a vertex v of a connected graph G is the distance between v and a vertex farthest from v in G. If $e(v) = \mathrm{diam}(G)$, then v is a *peripheral vertex* of G. The following is a consequence of the proof of Theorem 2.2.

Corollary 4.1. *Let G be a connected graph of diameter 3 or more. If G contains an end-vertex that is a peripheral vertex of G, then G is an irregular domination graph.*

By Corollary 4.1, if G is a connected graph with $\mathrm{diam}(G) = 3$ having a peripheral vertex of degree 1, then G is an irregular domination graph. This is also true for all connected graphs of diameter 4. We know of no connected graph of diameter 4 or more, however, that is not an irregular domination graph. By Corollary 4.1, if G is a connected graph of diameter 4 or more that is not an irregular domination graph, then no end-vertex of G is a peripheral vertex of G. We close by stating the following conjecture.

Conjecture 4.1. *Every connected graph of diameter 4 or more is an irregular domination graph.*

Acknowledgment

We are grateful to Professor Gary Chartrand for suggesting the concept of irregular domination graphs to us and kindly providing useful information on disconnected irregular domination graphs.

References

[1] A. Ali, G. Chartrand, P. Zhang, *Irregularity in Graphs*, Springer, New York, 2021.
[2] A. Ali, G. Chartrand, P. Zhang, On irregular and antiregular domination in graphs, *Electron. J. Math.* **2** (2021) 26–36.
[3] P. Broe, G. Chartrand, P. Zhang, Irregular domination in trees, *Electron. J. Math.* **1** (2021) 89–100.
[4] P. Broe, G. Chartrand, P. Zhang, Irregular orbital domination in graphs, *Int. J. Comput. Math. Comput. Syst. Theory* **7** (2022) 68–79.
[5] G. Chartrand, P. Zhang, A chessboard problem and irregular domination, *Bull. Inst. Combin. Appl.*, To appear.
[6] T. W. Haynes, S. T. Hedetniemi, M. A. Henning, *Domination in Graphs: Core Concepts*, Springer, New York, 2022.
[7] C. Mays, P. Zhang, Irregular domination graphs, *Contrib. Math.* **6** (2022) 5–14.