

## Autobiographical notes\*

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## School life

I was born in Sault Ste. Marie, Michigan, in 1936 (the same year that the first book on graph theory, written by Dénes Kőnig, was published). As a youngster, I attended many movies with my parents. In fact, my parents started taking me to movies when I was three years old. My father especially liked movie musicals, which I did too. When he was young, he played the piano in movie theaters for silent movies. (I remember very often, after coming home from watching a musical at the movies, my father would play some of the songs he just heard on the piano.) Having lived through the depression, he worked hard to support his family. Through him I learned the importance of accomplishing something, contributing to society, and not doing something just for money.

I was especially interested in comic books when I was a kid. My parents would often buy comic books for me. Because of that, I learned to read before I went to Kindergarten. When I thought the stories could have been better, I started writing my own comic books but I was terrible at drawing. I lived in Sault Ste. Marie until I was 13 years old, when my parents moved to Lansing, Michigan. I went to Lansing Sexton High School. I had some great teachers there - I especially remember Gertrude Benson (Mathematics) and Marjorie Ludwig (English). Also, I got to know the Assistant Principal (Elizabeth Lawry) very well. My favorite subject there was English, followed by Mathematics, but I also enjoyed Chemistry and History.

One important event for me in high school had nothing to do with my classes. I lived close to the high school and I always walked to and from school. I would pass by the high school auditorium on the way home. One day, when I was a senior, I heard music coming from the auditorium on my way home. I opened the door, walked in, and sat down. I learned that a touring company of the musical “South Pacific” would be performing in the high school and the performers were rehearsing. I had never seen a live musical before. I bought a ticket for the show and was fascinated by it. I’ve been hooked on musicals ever since. I have probably been to about 800 stage musicals now.

Another interesting high school memory did have something to do with my classes. During my last semester of high school, my mathematics teacher was Mr. Fox and the course was trigonometry. At the beginning of each class, he had some record-keeping to do, as every teacher had. When we came to class, he would often have a mathematics problem on the board (not trigonometry). These problems were often intriguing. We were supposed to see if we could solve them (which was not done very often) and whether this suggested any other problems to us. I found these problems fascinating and started learning the importance of asking questions.

## Bachelor’s degree

Miss Benson wanted me to go to the University of Michigan, where she went, but all of my friends were going to Michigan State University (called Michigan State College at the time). Also, my parents really didn’t have the money for me to do anything else. I was fortunate to receive a tuition scholarship and in the Fall of 1954, I went to Michigan State. No one in my family (father, mother, aunts, uncles) had finished high school but I was going to college. My parents always emphasized college and working hard, however. When I went to Michigan State, I was supposed to choose a major, which I knew nothing about. My father suggested majoring in engineering so I could get a job when I graduated. I saw that there was such a thing as chemical engineering. So, I was essentially a chemical engineering major for two years. I had a terrible schedule. My classes would often start at 8:00 each morning and I wouldn’t get out many times until 6:00. Michigan State was in East Lansing but I lived with my parents in Lansing. I never had a car so getting to the university and getting home was not easy. Although I had a tuition scholarship, I had no

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\*Based on an interview conducted by Akbar Ali

money. I recall that my parents would give me the money I needed for textbooks. (I always bought new textbooks, not used, because I would often wear out the books. Also, I would never sell them later.) I was also given \$6 a week for lunch and bus fare. I was required to take ROTC (Reserve Officers' Training Corps) as a freshman and sophomore. I needed to wear a uniform once a week because we marched once a week. I had never shot a rifle before. (I had never even held one.) I didn't like that either but I grew to like ROTC when we were studying map reading and military strategy. All I ever seemed to be doing was studying and riding on a bus, however - and I wasn't enthusiastic about my major.

Near the end of my sophomore year, I realized that the courses I really enjoyed were mathematics courses and I decided to switch majors. So, as I entered my junior year, I had become mathematics major. This changed everything for me. I was really enjoying going to college now even though I hadn't taken the mathematics courses that other junior mathematics majors had taken. The students taking these mathematics courses enjoyed mathematics just as I did. I was now doing well in every course I was taking. When I was a junior, I joined the local chapter of the mathematics honorary Pi Mu Epsilon and never missed a meeting even though they took place in the evening (and I had no car). At these meetings, I got to meet other mathematics students and other faculty members in the Department of Mathematics. I even got to meet the Chair of the Department of Mathematics (J. Sutherland Frame), who was so enthusiastic about mathematics. At one Pi Mu Epsilon meeting, Professor Frame asked me what I was going to do after I graduated. I told him that I had no idea. He suggested that I stay on to become a graduate student. He said that I could become a graduate assistant. This sounded like a great idea. In June of 1958, I graduated with a Bachelor of Science degree, with a major in mathematics and minors in physical science and foreign languages. That summer my father got me a job with the telephone company.

## **Master's degree**

In the Fall of 1958, I was a graduate student at Michigan State, working on a Master's degree in mathematics. My Master's program didn't get off to the start I had hoped for. The fact that I didn't have the background mathematics courses that other mathematics majors had as an undergraduate was a negative for me. At the beginning, I was taking three courses and running two recitation sections each semester. But, during my second year as a graduate student, I took only two mathematics courses each semester and I was teaching two courses on my own. I surprised myself how much I enjoyed teaching and was learning from the courses I taught. Things settled down for me that year and now I was doing well in everything. I received my Master's degree in 1960.

## **Ph.D., graph theory and the first job**

Even though I still didn't have great confidence in myself, I decided to try for a Ph.D. in mathematics. I continued taking two courses each semester and teaching two of my own. I knew Professor Frame and the new Department Chair, Professor Charles Wells, quite well and I kept asking for new courses to teach. They always would let me do this. Just a few years earlier, Michigan State obtained a computer. (It seems strange now but at that time the university had only one computer, called MISTIC. It was a gigantic computer housed in an air-conditioned room.) When I was an undergraduate, one of the topology professors (Professor John Hocking) taught a course from it and I took that course. There was only one programming course at the university (it was in machine language) and it was taught out of the Department of Mathematics. Late one academic year, Professor Wells came to me and said that no faculty member wanted to teach that course the following year. He knew that I had taken the course as an undergraduate and wondered if I'd be willing to teach it the following year. Since there was no textbook for the course, I would have to make up the course myself. This sounded like great fun, I agreed to do it, and ended up teaching it four times. (At the end of my first year on my doctoral program, I had finally saved enough money to put a down payment on a car.)

Even though I was doing well in my courses, I felt that most of my time was spent doing assignments and preparing for exams. I never felt that I had the understanding of the material that I needed. Starting with my second year on my doctoral program, I decided to change the way I was studying. When I was assigned homework problems, I started investigating what results would be changed if the hypothesis was altered. Also, what other conclusions could one obtain with the same or altered hypotheses? This greatly helped my studying. I also felt that I was getting a better feeling for the subjects. Late in my second year of my doctoral program, I found a note in my mailbox from the Head of Graduate Programs, Professor Bonnie Stewart. He wanted to see me. This made me very nervous. What did I do wrong? I made an appointment to see him. He said that I was completing my second year on my doctoral program and have never indicated what I might specialize in. I told him that while I enjoyed almost every course I had taken,

I never saw any subject that I felt I could contribute to. He suggested that I take a reading course in something else the following Fall semester. Professor Leroy Kelly agreed to give me a reading course in functional analysis. During the summer I took a course in ring theory. Professor Stewart was the instructor. There is one exam he gave I still remember. I completed the exam early but I never left any exam in any course early. I went through the exam again and in one of the problems, I thought I could prove something more general than what was being asked. I gambled and decided to prove the more general result, which left the exam problem as a corollary. When I got the exam back, Professor Stewart was clearly very pleased with what I had done. I felt that I was getting better all the time with mathematics. The following Fall, I took the reading course in functional analysis, but I wasn't excited about it.

When I became a graduate student, Professor Frame told all graduate assistants that we should attend all colloquium talks. Since I was practically living at the university, that was easy for me to do. During that Fall semester 1962, I attended a colloquium talk given by Professor Edward Nordhaus. The talk was in an area of mathematics I had never seen before: Graph Theory. I was really excited about this. The very next day, I visited Professor Nordhaus, asking him if I could take a reading course in graph theory from him the next semester (Winter 1963). He told me that the only book he knew in graph theory was in German but he said that he could give me a reading course in lattice theory. I asked him to let me think about this. Soon afterwards, I saw in the Notices of the American Mathematical Society an advertisement for a book in graph theory - written in English by Oystein Ore of Yale University. I ordered this book immediately. When I received the book, I noticed that Professor Nordhaus' name was mentioned in it. I showed this to Professor Nordhaus and he said, "Let's both read it." So, I was scheduled to take a reading course in graph theory during Winter semester 1963. I didn't wait until then, however. I spent much of the holiday break of 1962-1963 reading the book.

Only after the first two weeks of Winter semester 1963, I knew that graph theory was the subject for me and I asked Professor Nordhaus if he would be my doctoral advisor. I didn't know that he had never advised a doctoral student before. He agreed to be my advisor. He said that what I needed to do was to find a topic in graph theory to work on and find some problems on this topic. Because of the way I was now studying for the courses I took, I felt I could handle this. During that semester and the next semester (Spring 1963), I kept thinking of problems and was able to solve some of them. I put something in Professor Nordhaus' mailbox almost every day and I would receive a response the next day. I had several major worries though: Were the problems I was working on interesting enough? Were they appropriate for a doctoral thesis? Were they already known? I knew so little about graph theory. I went to the library often searching for information. I was working on my thesis about 40 hours a week at that time. Plus I was taking a course and teaching two courses but I enjoyed what I was doing.

The year 1963 was a challenging year as I required to take (and pass) the preliminary doctoral exams (just called the prelims). I had to take five exams (algebra, real analysis, complex analysis, graph theory, topology/differential geometry), all in a 10-day period. After several weeks, I learned that I had passed. Now I could concentrate on my thesis. There was one statement I was trying desperately to prove (if it was true), namely, that if  $G$  is a connected graph that is not a path and one continued to take the line graph of the resulting graph, then eventually the resulting graph would be Hamiltonian. In fact, I gave up the opportunity to teach during the summer of 1963 so I could continue to work on it. I spent day after day on this without success. However, after Fall semester started, I thought of a proof. (Sometime later I found a much simpler proof.)

During Fall 1963, I was convinced that I would be able to complete my thesis and earn my Ph.D. in June of 1964. I was even thinking about what would happen after that. I never thought that I would enjoy teaching as much as I did but now I wanted to be a faculty member someplace. So, during Fall 1963, I decided to look into finding a faculty position for Fall 1964. I knew almost nothing about other universities and certainly had not visited any. I decided to write to seven universities nearby I had heard of. I didn't even know if they had a position open for a mathematics instructor. I remember going to the post office one morning and mailing out seven letters. One of the places I wrote to was Western Michigan University. The very next afternoon, I received a phone call at home from James Powell, the Chair of the Department of Mathematics at Western Michigan, saying that he had received my letter and wanted to invite me for an interview shortly after a mathematics conference that was to take place in early January. I thanked Professor Powell for calling me and told him that I would go there for the interview. Eventually I would hear from one other place I wrote to. The other five never responded. Some time later, I happened to mention to Professor Nordhaus that I had written to seven colleges and universities, and told him that I would have an interview in January at Western Michigan University. He was clearly surprised to learn what I had done and that I had agreed to interview at Western Michigan University. I never knew I was supposed to talk to him about this first and ask him for a letter of recommendation. I never did ask him for a letter of recommendation. Shortly afterwards, Professor Nordhaus told me that he had arranged a 1-year postdoc for me at the University of Waterloo. I knew nothing about that place and

had no interest in spending a year in Canada. I thanked him but told him I wasn't interested.

Sometime later, Professor Nordhaus talked to me again about the next year for me. He asked me if I was aware of the name Frank Harary. I told him that I had run across his name many times in the library. He told me that he knew Professor Harary, had called him, and there might be a possibility for a 1-year postdoc at the University of Michigan. If I was interested in that, Professor Harary would like to meet me. Actually, I never knew Frank Harary was a professor at the University of Michigan but I told Professor Nordhaus this did interest me. Professor Nordhaus set up a meeting in Ann Arbor for Professor Nordhaus and me to meet Professor Harary. I recall that this took place in October 1963. I needed to take what I written on my thesis so he could see how I wrote. So, Professor Nordhaus (he drove) and I went to Ann Arbor to visit Professor Harary. I thought he was a very unusual person but I liked him instantly, even though he made me a bit nervous. His office was a mess. He looked at my thesis and I remember him saying, "You'll do." He told me that he was going to be working on a book and needed some assistance. He said that he had applied for a research grant. In the proposal, he had asked for money for one postdoc. If the grant came though, the postdoc was mine. He asked me if I would be interested and I said, "Definitely." He said that he was optimistic about the grant being approved but it wasn't a sure thing. In January 1964, I had heard nothing about the postdoc at the University of Michigan and so I went to Western Michigan University for the interview. I was never asked to give a talk. I never knew that people being interviewed normally gave talks. I met the Chair, Professor Powell, and several faculty members. One in particular I remember meeting was Professor Yousef Alavi. I thought everyone there was very nice but the university seemed so small. I met Professor Powell one last time during my interview visit before I returned to Lansing. He said that he would like to offer me a position there for Fall 1964. I asked if I could think about it and he said "yes". After a few weeks, I still had heard nothing about the University of Michigan but I felt that it wasn't fair not to respond to the offer from Western Michigan University. I called Professor Powell to tell him that I was turning down the offer.

It wasn't long after that when I learned from Professor Harary that the grant didn't come through for the following year (1964-1965), as he had hoped. So, now I had nothing for Fall 1964. I think it was May 1964 when I happened to run into a professor I had met at Western Michigan University who was visiting Michigan State University. He knew I had turned down their offer and asked me where I was going the following Fall. I told him what had happened. He told me that he believed everyone who had been offered the position at Western Michigan University had also turned down the offer. He said that if I was interested, perhaps I should call Professor Powell, which I did. He told me that the position was still open and if I wanted it, it was mine. He said the annual salary was \$8700 but he might be able to raise it to \$9000. I accepted the position but I told him I would accept the \$8700. I told him that I had not even earned the \$8700 yet. During my last year at Michigan State, Professors Nordhaus and Kelly arranged to have Professor Kelly's doctoral student in geometry (David Kay) and me work on a research project that combined graph theory and geometry. A paper on this project was written, submitted, and accepted for publication. That was my first paper but I never felt that I contributed much and certainly the idea for the project was not mine.

In June of 1964, I received a Ph.D. in mathematics from Michigan State University. After the ceremony, my parents arranged a reception for me. Attending the reception was my high school mathematics teacher Miss Benson and the Assistant Principal from high school, Miss Lawry. I was very happy about that, although I wish my English teacher Miss Ludwig had also been invited. My father told me that they did not invite Professor Nordhaus because they thought everyone there would be uncomfortable with such a highly educated person present. I stayed at Michigan State during the summer of 1964. Some of the students I knew there (both graduate students and undergraduates) had now heard about graph theory - because I was talking about it so much. I asked Professor Wells, current Chair of the Mathematics Department, if I could teach a course in graph theory that summer. It wasn't necessary to pay me. He told me I could but he would pay me. At the same time, he asked if I would be willing to teach an undergraduate course in set theory. So I taught two courses that summer. I made up the graph theory course and an undergraduate, whom I knew well, typed up the notes each day. That was the first course ever taught in graph theory at Michigan State.

## **Initial years at Western Michigan university**

After I accepted the position at Western Michigan University, I called Professor Harary to tell him what I had done. He told me that he would be running a Graph Theory Seminar during the coming Fall semester (Mondays at 4:00 if I recall) and that I would be welcome to attend. I told him I would do that. I contacted Professor Powell at Western Michigan University, telling him of my plans, and he said that he would arrange my teaching schedule so I could attend the seminar. I enjoyed going to the seminar in Ann Arbor and met several of Professor Harary's doctoral

students (Lowell Beineke, Steve Hedetniemi, Mike Plummer, Ed Palmer). All were very nice - and clearly very smart.

Although I never really knew what other faculty members were doing at Western Michigan, I had a strong feeling that very few were doing any research. But I enjoyed research and the only thing that hurt me was my background in graph theory. I was willing to work hard but I needed to know more. Certainly no one else was working in graph theory at Western Michigan. I don't think that they had ever heard of that area, which was not surprising as I had just heard of it two years earlier myself. But I think my enthusiasm for graph theory was starting to rub off, not only on some of the graduate students there, but also on some of the faculty members. I used my free time to write up papers that came from my thesis. I wrote three of these, one on connectivity and two on Hamiltonian line graphs. It wasn't all that exciting writing up papers by myself, so I decided that I would ask some friends from Michigan State to join me on other research I had done on my thesis. That was much more interesting. Doing research by myself was not fun.

## **Postdoctoral research with Frank Harary**

About two months into Fall semester 1964, Professor Harary told me that the research grant he applied for had come through for the following year (1965-1966) and that there was money for a postdoc and that I could have this if I wanted it. It paid \$7200 for 12 months but the money meant nothing. I accepted it immediately and the very next day I went to Professor Powell's office, telling him that it would be necessary for me to resign at the end of the academic year. He said that this would be good for me but it wasn't necessary to resign. I could have a leave of absence for 1965-1966 and return in 1966. I had never heard of a leave of absence but I took it.

I would occasionally drive back to Michigan State and talk with other graduate students. Professor Nordhaus had accepted a second student, Mehdi Behzad, in graph theory and I became friends with him. During the summer of 1965, I moved back to Lansing and spent nearly every day visiting Michigan State. I visited Professor Harary one day and Mehdi Behzad asked me if he could come with me, which he did. Professor Harary asked Mehdi what he was working on and he said a type of coloring he was calling total colorings (coloring vertices and edges). Professor Harary said instantly that this was the same as coloring the vertices of another graph - and the total graph was born.

I was looking for a place to live in Ann Arbor and it became clear that apartments there were much more expensive than I was accustomed to. During the Summer of 1964, I often played baseball at Michigan State with some graduate students and faculty members. (In fact, I spent many summers in high school and as an undergraduate at Michigan State playing baseball.) A graduate student playing baseball that summer was Al Polimeni, who was working in algebra. He got his Ph.D. in 1965 and was going to the University of Michigan on a 2-year postdoc. We agreed to share an apartment in Ann Arbor for 1965-1966.

I arrived in Ann Arbor during August of 1965 and visited Professor Harary right away, telling him that I was ready to work. He was working on a textbook in graph theory and needed some assistance. He wanted one-word titles for each chapter as much as possible. He knew what he wanted to include in each chapter. I knew the terminology and notation he was using. For the most part, my job was to find the original paper where a certain result appeared and rewrite the proof so that it was clear and written in Professor Harary's notation and terminology. Sometimes that wasn't so difficult but, more often than not, it was. (I was to learn something I already knew - although many of the results obtained were interesting and potentially important, the writing was often terrible.) Very few papers were written clearly and carefully. I located a number of errors. I didn't have much to do with the exercises that were chosen. Needless to say, I learned a lot by doing this.

The very day I went to Professor Harary's office in August 1965 to learn what he wanted me to do, he told me that he would soon be attending a conference at Cornell University and asked me if I would like to go with him. (I had never been to a conference before. In fact, when I was at Michigan State, I never knew that there was such a thing as a mathematics conference.) I told Professor Harary that I would like to go. He responded, "Good, you're driving." So I drove to Ithaca, New York with Professor Harary. He wanted to visit Niagara Falls, which we did. (The only time I've been at Niagara Falls was with Professor Harary.) When I returned from Cornell, it was time to get to work. Professor Harary had two offices, in fact two positions, one with the Institute of Social Research and the other with the Mathematics Department, so I had two offices as well. During 10:00-11:00 every weekday morning, he would meet with his doctoral students as a group at the Institute for Social Research and he asked me to join in. I found this fascinating as everyone (especially Professor Harary) discussed possible research projects. He would often take a research paper from the library and ask: What is another question the author didn't ask? It was extremely important to him that everything is written clearly. When the writing bothered him, he would often say, "So that's what he meant. Then why didn't he say so?" It would really annoy me when something I gave him contained an error

or was not written clearly enough. But he was very kind to me and would say, "Not to worry." He treated me like his own doctoral student. I went with him and Mike Plummer to a graph theory conference at the University of Waterloo. There I met William Tutte and Crispin Nash-Williams. I would meet many famous graph theorists through Professor Harary.

Near the end of my 12 months at Ann Arbor, Professor Harary said that it's time for the two of us to write a paper together. He brought up a question on connectivity, which I was not only able to answer but came up with another result to add to the paper. Then he defined the concept of permutation graph to me and asked me to find which permutation graphs of cycles were planar. In the process of working on this, I told him that it would be useful to have a concept for a graph that could be embedded in the plane where all vertices were on the boundary of the exterior region. He gave me three choices of what to call this. Two were "outplanar" and "outerplanar". I chose the second; it sounded better to me. So I had a second paper with him.

One thing that was always important to Professor Harary is that the problems one worked on should be interesting and any resulting paper should be clear and well written. He also emphasized giving clear and interesting talks. I remember attending a conference with Professor Harary where we sat together during a lengthy sequence of graph theory talks. After each talk, he would tell me what he liked and didn't like about each talk. I agreed with him on everything. When one gives a talk (and even when one writes a paper), the speaker (writer) should be telling a story, explaining how the subject fits in with what is already known. It can be boring and tiring to sit in on one talk after another, which makes it all the more important to make the talk as interesting as possible.

## **Years after postdoctoral research**

I really never thought I would return to Western Michigan after my postdoc. I was having too much fun learning and doing research but I never took the time to look for another position. Then I got a phone call from Professor Powell, saying that he hoped I was returning and that my salary the next year would be \$12,000. I then thought: Well, I'll spend one more year at Western Michigan University. So, in the Fall of 1966, I returned to Western Michigan. I was really missing working on research with others during that Fall semester. Two friends of mine had moved to California, one in Berkeley and one near Los Angeles. Each told me that I should visit them. So, I flew out to San Francisco during Christmas vacation for a one-week vacation - the first vacation I had since 1949 (when my parents drove me to Detroit to see a baseball game). My friend and I did many things in San Francisco. Then we flew to Los Angeles to meet my other friend. We toured Los Angeles and went to Disneyland. We flew to Las Vegas for one day. My Berkeley friend had to return but I had one day left on my vacation, so the other friend and I drove to Santa Barbara.

When I returned to Western Michigan, I was back to a place with no one else to talk about graph theory. I decided that I had enough and I told the Chair that I would be resigning at the end of the year. He asked me what I would be doing and I said I didn't know. A former graduate student I knew was at Binghamton, New York and he told me that I might be able to get a position there for 1967-1968. I flew there for an interview. But evidently a one-year position never materialized there. Then the Chair asked: Why not have the person from Binghamton visit Western Michigan for the next year and hire someone to work with me at the same time? This is exactly what happened. I didn't know any person in graph theory we could hire but Shashi Kapoor from Michigan State was getting his Ph.D. in topology and I knew he was interested in graph theory. Western Michigan hired him. So, when Fall semester of 1967 came, there were two people to work with. Also, I was promoted to Associate Professor, which came as a surprise as I had never expressed any interest in being promoted. When I was interviewed by Western Michigan, I was told that the plan was for the Mathematics Department to have a doctoral program in 1968. I learned that this was still on schedule and some of the graduate students were planning to stay on to work on a doctoral degree. There were graduate students now working in graph theory as well.

Starting in 1967, there were now groups of faculty and graduate students working on graph theory problems. In addition to Shashi Kapoor, faculty members Yousef Alavi and Don Lick joined in. We were not only accomplishing research but I think everyone was enjoying this. I continued my connections with Professor Harary. He said that his grant would allow me to spend the summer of 1967 at the University of Michigan, which I did. While Professor Harary was gone much of the summer, I did research that summer with Steve Hedetniemi and Dennis Geller from the University of Michigan. I enjoyed all the research I was involved with. During the following academic year (1967-1968), the Chair of the Mathematics Department suggested that Western Michigan University should host a graph theory conference of its own. I thought this was a great idea and Shashi Kapoor agreed to join me to direct a conference. Later Yousef Alavi joined in. A conference took place in November of 1968 and this would be the first of

nine such conferences, held every four years. Alavi became one of the main directors of all these conferences. In 1969, the Mathematics Department at Western Michigan graduated its first doctoral student. At that time, I had a doctoral student of my own (John Mitchem), who received his doctoral degree in 1970 and obtained a faculty position at San Jose State University. I was very fortunate to have a nice, bright, hard-working student as my first of eventually 22 doctoral students.

Later Western Michigan University hired another faculty member, Arthur White, who worked in topological graph theory. I was surprised to learn that I was promoted to Professor in 1970, as that topic had never come up. We were working on all kinds of topics in graph theory. I enjoyed this, as I always wanted to continue to learn. Graph theory courses had been developed and I was also working on textbooks, as I always enjoyed writing. I was attending conferences and keeping in contact with Professor Harary.

After my one-day visit to Santa Barbara in December of 1966, I had learned that there was a faculty member (Paul Kelly) at the University of California Santa Barbara whose name I had run across in connection with a famous problem in graph theory (the Reconstruction Problem). I thought of taking a sabbatical leave there in 1970-1971. I contacted Kelly and he agreed to have me spend the year there. It turns out that Kelly really didn't do much graph theory anymore, which was a disappointment to me. But Seymour (Sy) Schuster was taking a sabbatical there at the very same time. We became friends and did research together there, so the sabbatical leave was a bit of a success after all.

## Writing books

As far as books are concerned, I always enjoyed writing. I thought that I could write clear proofs and make the subject interesting. I've always been interested in graph theory history and the origin of various concepts, topics, and problems. Western Michigan University had a graduate sequence in graph theory, so I felt it was a good idea to write a book on the subject. I asked my friend Mehdi Behzad to join me and we wrote "An Introduction to the Theory of Graphs" published in 1971. Later, I wanted to write a graph theory book describing what I felt was the fun applications of graph theory. I did this by myself, called "Graphs as Mathematical Models" published in 1977.

I didn't really know how many books existed worldwide when I thought about writing a book on graph theory. Of course, early on there was König's book (in German), a book by Claude Berge (in French), one by Zykov (in Russian) and Ore's book. Also, there was Harary's book. There were several special topics books, including books by Tutte, Moon, Nebesky, Ringel, Seshu and Reed, Ford and Fulkerson, Grossman and Magnus, and Busacker and Saaty. Many more came later in the 1970s. But, when I started thinking of writing a book, I only knew half of these books at most.

## Journal of graph theory

A major event for me occurred in 1974, when I received a phone call at home from Professor Harary. He told me that he had been on the telephone with Wiley Publisher and he had suggested starting a new journal (the Journal of Graph Theory) and Wiley was interested. He told me that he would only go ahead with this if I agreed to be its managing editor. With teaching, doing research, writing books, and directing doctoral students, I was extremely busy but this sounded like great idea and I always enjoyed working with Professor Harary, so I immediately agreed to do this.

At first, it was difficult finding mathematicians interested in submitting papers to this new journal. I suspect this is the same fate that every new journal experiences. Doing this became very time-consuming. And I never had released time for this. Since this was long before email, I wrote (and typed myself) many letters and made many phone calls. The postage bill at Western Michigan University was very high for the journal. The first edition of this new journal was not published until 1977, however. A great deal of effort and time went into getting the journal started. When a paper was submitted, I made three copies of it, so I had four copies altogether. I sent out a letter to the author immediately, telling him/her that I had received the submitted paper and thanking him/her for thinking of the journal.

At the beginning, I met regularly with Professor Harary, usually in Ann Arbor but sometimes in Marshall, Michigan, halfway between Ann Arbor and Kalamazoo. Meeting in Marshall was more productive because when I met with Professor Harary in Ann Arbor, the phone was ringing constantly. People were always calling him. I had a copy of each paper submitted with me since the previous time I had met with him. We would go over each paper, determining how interesting the paper was. We agreed much of the time. Professor Harary also checked how well written it was. If, in his opinion, it was not written well, he didn't even want it refereed. He felt it was wasting a referee's time and also that of a potential reader. For most papers, it was decided to have them refereed by two people. I remember him



saying that well-known graph theorists would not spend much time to evaluate a paper, only perhaps giving their opinion on how interesting it was. Other referees might be willing to read it in more detail. That's all that could be hoped for. Other than that, the authors had to be trusted. I wrote to two referees for a paper, requesting them to referee it within three months. Some answered that they would do it, a very few said they couldn't do it, and many never responded. If after three months, I didn't hear from a potential referee, I wrote a reminder, asking if he/she could finish it in the next month. At the same time, I wrote to the author, informing him/her that I had written to the referee and apologizing for taking so long. So, over the years, I wrote an enormous number of letters. (I wish that email had existed at that time.). It reached a time when I was spending about 20 hours a week on the journal, while at the same time teaching two courses, doing research of my own, directing a doctoral student, and working on a book. But I enjoyed working with Professor Harary and felt that I was doing something worthwhile and learning more about graph theory.

Professor Harary thought it would be interesting for those in graph theory to have the journal include survey papers on certain topics as well as articles about mathematicians who contributed to the history of graph theory. I found this quite enjoyable myself. This was another emphasis of the Journal of Graph Theory. Indeed, I received letters from readers telling me that they enjoyed these articles. Of course, this was not my idea but Professor Harary's. Eventually, the editorial office moved away from Western Michigan University and those in charge of the journal didn't like this idea - too bad! Professor Harary then thought of introducing another journal that would emphasize this. He wanted to call it the Graph Theory Monthly. I told him that it would need too much effort to manage a journal that came out every month. The idea then changed to the Graph Theory Quarterly. Discussions continued on this for sometime but I retired from Western Michigan University on January 1, 2005 and Professor Harary passed away on January 4, 2005 (only three days apart).

## Closing remarks

Despite the fact that I retired in 2005, I still enjoyed working on research and writing books. In 1996, the Department of Mathematics at Western Michigan University hired a new faculty member, Ping Zhang, who had the same degree of enthusiasm about research and writing as I had. This resulted in collaborations on many projects with her. Indeed, she became an important friend of mine. We have now co-authored over 100 research papers and many books.

As I now look back at 55 years of being a mathematician, I have several observations. I was happy, indeed fortunate, that I chose to work in graph theory. Like others have stated, doing research is the fun part of being a mathematics faculty member. I have worked with over a hundred mathematicians on many projects covering many areas of graph theory. I'd like to think that my main contributions to mathematics may have been (1) creating topics in graph theory which I (and hopefully others as well) found interesting and (2) describing results and topics in a way that is clear and interesting to others. However, I always enjoyed teaching. I spent several years chairing the Graduate Committee, which I also enjoyed. In that position, I had the opportunity to meet with every graduate assistant every semester and I visited the classes of each one (many of whom were excellent teachers already) and spent hours going over the one hundred or more applications for assistantships for the following year.

There are many in the field of graph theory I read about whom later I had the wonderful opportunity to meet. In addition to Frank Harary and Paul Erdős, I've met Ron Graham, Fan Chung, Vera Sós, Claude Berge, William Tutte, Kazimierz Kuratowski, Hassler Whitney, Roberto Frucht, Wolfgang Haken, and Gerhard Ringel; I've talked with Karl Menger on the phone; and corresponded with Kenneth Appel, Alexander Zykov, Vadim Vizing, and Nicolaas de Bruijn. Then there are others I have worked with and/or have gotten to know in the area of discrete mathematics who are both very nice and very talented. Among those on this long list are Ralph Stanton, Robin Wilson, Lowell Beineke, Steve Hedetniemi, Mike Henning, Teresa Haynes, Bob Brigham, and Arthur Benjamin. How fortunate I have been.

## Some selected papers

At the end of this document 45 selected papers of some 300 that I have authored or co-authored are listed, under the heading "References". In what follows, a sentence or two about each of these papers is/are given.

[45] is my first paper and only paper as a graduate student; [7] is my first paper by myself and first paper from my doctoral thesis; [6] is a paper from my thesis on connectivity; [16] is my first paper with Frank Harary and in this paper outerplanar graphs are introduced; [15] is a paper that has been often referenced; [31] is a paper with my first doctoral student about well-known bounds named after my advisor; an improved theorem from my thesis with a



better proof is given in [41]; the papers [22, 24, 43] are about some interesting properties of the Petersen graph; [38] is the first of several papers on distance between graphs; [33] is a paper with my advisor; highly irregular graphs are introduced in [2]; [1] is concerning a conjecture that Paul Erdős offered \$5 for (still unsettled); A variety of ways to define an irregular graph is given in [10]; Petersen's theorem looked at in another way in [4]; the irregularity strength is introduced in [26]; Steiner distance in graphs is introduced in [34]; [23] is a paper with an applied flavor; [25] is about a new look at distance; the partition dimension of a graph is introduced in [39]; geodetic sets are discussed in [17]; [11] is a fundamental paper on metric dimension; the hull number of a graph is introduced in [19]; radio labelings of graphs are introduced in [12]; geodetic numbers of graphs are discussed in [18]; the paper [44] is about the Steiner numbers of graphs; the convexity number of a graph is introduced in [42]; [14] is a paper with mathematicians who attended the last Kalamazoo Quadrennial Conference; boundary vertices are introduced in [13]; the paper [20] is about a new framework for domination; another way to look at Hamiltonian walks is given in [40]; Hamiltonian colorings are introduced in [32]; rainbow connection is introduced in [27]; rainbow connectivity is introduced in [28]; another way to look at connectivity is given in [35]; the paper [37] is about a variety of complete colorings; the sigma chromatic number of a graph is introduced in [36]; a four colorings theorem (not the Four Color Theorem) is discussed in [21]; the paper [30] is concerned with a story problem; the papers [8, 29] are about Eulerian graphs; looking at Ramsey numbers in other ways in [3, 5]; the paper [9] is about rainbow disconnection.

## List of books

- *Introduction to the Theory of Graphs*, Allyn and Bacon, 1971 (with M. Behzad).
- *Graphs as Mathematical Models*, Prindle, Weber & Schmidt, 1977.
- *Introductory Graph Theory*, Dover Paperbacks, 1985.
- *Applied and Algorithmic Graph Theory*, McGraw – Hill, 1993 (with O. R. Oellermann).
- *Introduction to Graph Theory*, McGraw – Hill, 2004 (with P. Zhang).
- *Chromatic Graph Theory*, CRC Press, 2008 (with P. Zhang).
- *Discrete Mathematics*, Waveland Press Inc., 2011 (with P. Zhang).
- *A First Course in Graph Theory*, Dover Publications Inc., 2012 (with P. Zhang).
- *The Fascinating World of Graph Theory*, Princeton University Press, 2015 (with A. Benjamin and P. Zhang).
- *Graphs & Digraphs*, Sixth Edition, CRC Press, 2016 (with L. Lesniak and P. Zhang).
- *Mathematical Proofs: A Transition to Advanced Mathematics*, Fourth Edition, Pearson, 2017 (with A. D. Polimeni and P. Zhang).
- Forthcoming book: *How to Label a Graph*, Springer, 2019 (with C. Egan and P. Zhang).
- Proposed new edition: *Chromatic Graph Theory*, Second Edition, CRC Press, 2020 (with P. Zhang).

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